Reviving trinification models through an E_6-extended supersymmetric GUT

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We present a supersymmetric (SUSY) model based on trinification $[\mathrm{SU}(3)]^3$ and family $\mathrm{SU}(3)_\mathrm{F}$ symmetries embedded into a maximal subgroup of E_8, where the sectors of light Higgs bosons and leptons are unified into a single chiral supermultiplet. The common origin of gauge trinification and of the family symmetry from E_8 separates the model from other trinification-based GUTs, as it protects, in particular, the Standard Model fermions from gaining mass until the electroweak symmetry is broken. Furthermore, it allows us to break the trinification symmetry via vacuum expectation values in SU(3)-adjoint scalars down to a left-right symmetric theory. Simultaneously, it ensures the unification of the gauge and Yukawa couplings as well as proton stability. Although the low-energy regime (e.g., mass hierarchies in the scalar sector determined by a soft SUSY-breaking mechanism) is yet to be established, these features are one key to revive the once very popular trinification-based GUTs.

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I. INTRODUCTION

Finding a compelling theory for the unification of the fundamental interactions that is capable of reproducing known features of the Standard Model (SM) has been a major goal of the theoretical physics community. Popular SM extensions are supersymmetric (SUSY) grand unified theories (GUTs) based on simple Lie groups such as, e.g., SU(5) [1], SO(10) [2], E_6 [3], and E_7 [4]. However, many of the existing GUTs typically suffer from various issues with, e.g., proton stability, fine-tuning, and hierarchies in parameters such as fermion masses and mixings lacking a fundamental explanation, as well as with inconceivably complicated parameter spaces severely reducing their predictive power.

GUTs inspired by E_6 are becoming increasingly popular due to their rich phenomenology and their many attractive properties (see, e.g., Refs. [5–8]). One such GUT scenario based upon a maximal rank-6 subgroup $[\mathrm{SU}(3)]^3 \subset E_6$ and known as gauge trinification (T-GUT) was initially proposed by Glashow in 1984 [9]. The trinification symmetry is typically identified as a left-right-color product group, i.e., $[\mathrm{SU}(3)]^3 \equiv \mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R} \times \mathrm{SU}(3)_\mathrm{C}$, and is supplemented by a cyclic permutation symmetry $\mathbb{Z}_3$ forcing the gauge couplings to unify, i.e., $g_\mathrm{U} \equiv g_\mathrm{L} = g_\mathrm{R} = g_\mathrm{C}$. One of the appealing features of T-GUT models is that all the matter fields, which belong to bitriplet representations (reps) of the trinification symmetry,

$$(\boldsymbol{L}^i)^l_r = \begin{pmatrix} \boldsymbol{H}_{11} & \boldsymbol{H}_{12} & e_\mathrm{L} \\ \boldsymbol{H}_{21} & \boldsymbol{H}_{22} & \boldsymbol{\nu}_\mathrm{L} \\ e^c_\mathrm{R} & \boldsymbol{\nu}^c_\mathrm{R} & \boldsymbol{\phi} \end{pmatrix}^i, \qquad \begin{aligned} (\boldsymbol{Q}^i_\mathrm{L})^x_l &= (\boldsymbol{u}^x_\mathrm{L} \;\; \boldsymbol{d}^x_\mathrm{L} \;\; \boldsymbol{D}^x_\mathrm{L})^i, \\ (\boldsymbol{Q}^i_\mathrm{R})^r_x &= (\boldsymbol{u}^c_{\mathrm{R}x} \;\; \boldsymbol{d}^c_{\mathrm{R}x} \;\; \boldsymbol{D}^c_{\mathrm{R}x})^{\top i}, \end{aligned} \tag{1}$$

can be embedded into three **27**-plets of E_6 as $\mathbf{27}^i \to (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1})^i \oplus (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{3})^i \oplus (\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})^i$. Here, the left, right, and color SU(3) indices are l, r, and x, respectively, while the generations are labeled by an index i (for an alternative realization containing the trinified gauge symmetry $[\mathrm{SU}(3)]^3$, see Refs. [10,11]). Some T-GUT versions claim to preserve baryon number naturally [12,13] but can also be engineered to account for the baryon-antibaryon asymmetry in the Universe through heavy Higgs decays at oneloop [14]. They can, in principle, accommodate any quark and lepton masses and mixing angles [12] while neutrino masses can be generated by, e.g., a radiative [13] or an inverse [15] see-saw mechanism. However, despite some progress in recent years, the T-GUT scenarios remain among the least explored extensions of the SM. One of the major theoretical challenges in building the SUSY-based T-GUTs is finding a stable vacuum with spontaneously broken gauge trinification while keeping a low number of free parameters at the GUT scale.

In order to avoid GUT-scale lepton masses, previous realizations of T-GUTs introduced either additional unmotivated Higgs multiplets [12,13,15–24], whose vacuum expectation values (VEVs) provide a consistent spontaneous symmetry breaking (SSB) of trinification down to the SM gauge symmetry, or higher-dimensional operators [15,17,18,20,25]. Such constructions may, however, result in severe phenomenological contradictions with proton stability [12,13,18] and too many unobserved low-scale signatures [9,17,22,23,25,26]. As a consequence, a large number of free Yukawa parameters in the superpotential has to be highly fine-tuned in order to reproduce the SM mass hierarchies [13]. A proper renormalization group (RG) analysis of a high-scale SUSY model containing a few hundreds of particles and couplings and accounting for several SSB scales down to the effective low-energy SM-like theory remains barely feasible in practice. Thus, deriving even basic features of the SM (such as fermion mass/mixing hierarchies and Higgs sector properties) as a low-energy effective field theory (EFT) limit of a T-GUT remains a big unsolved problem (for more details, see, e.g., Ref. [27] and references therein).

In this paper, we propose a new way to resolve the problem of GUT-scale masses of the SM leptons inspired by an embedding of the trinification $[\mathrm{SU}(3)]^3 \subset \mathrm{E}_6$ and family $\mathrm{SU}(3)_\mathrm{F}$ symmetries into the maximal exceptional symmetry group E_8. A common origin of family symmetry and SM gauge symmetries from $[\mathrm{SU}(3)]^3 \times \mathrm{SU}(3)_\mathrm{F} \subset \mathrm{E}_8$ implies that, in particular, the light Higgs and lepton sectors originate from the same (tritriplet) rep of $[\mathrm{SU}(3)]^3 \times \mathrm{SU}(3)_\mathrm{F}$. Having such light Higgs-lepton unification in the E_6-extended theory (inspired by E_8) leads to a complete unification of quark and lepton Yukawa couplings for all three generations (as well as the quartic interactions of the scalar potential) at the trinification-breaking scale. This is at variance with popular SO(10) and Pati-Salam models where the unification of Yukawa couplings is restricted to the third family [28–40]. Such a distinct feature of the high-scale model dramatically reduces its parameter space making its complete analysis computationally simple, at least at tree level. We have found that the proposed E_6-extended T-GUT model gives rise to an effective left-right (LR) symmetric theory with specific properties. The remnant $\mathrm{SU}(3)_\mathrm{F}$ family symmetry reduces the number of allowed terms in the LR-symmetric EFT, simplifying its matching procedure with the high-scale theory and making its RG flow analysis technically feasible. A consistent match of the LR-symmetric EFT with the SM at low scale would then strongly constrain the hierarchies in the soft SUSY-breaking sector, offering new possibilities for studies of the SUSY breaking in E_6-based theories.

II. E_8-INSPIRED FAMILY SYMMETRY

In earlier work by some of the authors [41], it was understood that the SM gauge group can arise dynamically from a non-SUSY T-GUT in a scenario where fermions and scalars belong to the same E_6 reps [augmented by a global $\mathrm{SU}(3)_\mathrm{F}$], thus hinting at a possible presence of SUSY at (or beyond) the GUT scale. In particular, the color-neutral scalars $\tilde{L}$ (containing the Higgs scalars) and fermions L (containing the SM leptons and right-handed neutrinos) could then be naturally considered as components of $\boldsymbol{L}$. Here and below, the notations $\tilde{f}$ and f for scalar and fermion components of the superfield $\boldsymbol{f}$ are used.

Inspired by this observation, the implications of a Higgs-lepton unification in a SUSY T-GUT were explored, with local gauge trinification $[\mathrm{SU}(3)]^3$ and global family $\mathrm{SU}(3)_\mathrm{F}$ motivated by a minimal E_6 embedding into E_8 as $\mathrm{E}_6 \times \mathrm{SU}(3)_\mathrm{F} \subset \mathrm{E}_8$ [42,43]. Indeed, such an E_6-extended trinification model inspired by its E_8 embedding can be considered as an approximation to the full gauge $[\mathrm{SU}(3)]^3 \times \mathrm{SU}(3)_\mathrm{F} \subset \mathrm{E}_8$ theory in those regions of parameter space where gauge $\mathrm{SU}(3)_\mathrm{F}$ interactions are suppressed, $g_\mathrm{F} \ll g_\mathrm{U}$. A special interest in E_8-based models originates from string theories where massless sectors are described by the $\mathrm{E}_8 \times \mathrm{E}_8'$ symmetry [43,44].

At variance with the non-SUSY model [41], incorporating the $\mathrm{SU}(3)_\mathrm{F}$ family symmetry in a SUSY T-GUT model with only tritriplets of $[\mathrm{SU}(3)]^3 \times \mathrm{SU}(3)_\mathrm{F}$ (specified in the first three rows of Table I) leads to a scalar potential containing flat directions with color-breaking VEVs. Even with the inclusion of soft breaking terms, such a model at tree level is necessarily inconsistent with the SM at low scales. Alternatively, the desired trinification SSB becomes possible in a SUSY T-GUT when relaxing $\mathrm{SU}(3)_\mathrm{F}$. However, this reintroduces GUT-scale masses for those SM leptons that are SUSY partners of the Goldstone bosons from $\tilde{L}$, due to terms such as $-\sqrt{2} g_\mathrm{U} (\tilde{L}_i^*)^r_{l_1} (T^a_\mathrm{L})^{l_1}_{l_2} (L^i)^{l_2}_r \lambda^a_\mathrm{L}$. These terms lead to gaugino-lepton mass terms of the order of the T-GUT-breaking VEV $\tilde{L}^i$. Although components in the trinification gaugino fields $\lambda^a_{\mathrm{L,R}}$ could in principle build up one generation of the SM leptons, we find such a construction unappealing both due to the reduction of the family symmetry and the abandonment of the full Higgs-lepton unification. Besides, the gaugino mass scale in this case would then be unnaturally small for a consistency with the SM lepton sector.

This gaugino-lepton mixing indeed posed a big problem for early attempts to consistently unify the Higgs and lepton sectors. However, rather than including additional copies of $\boldsymbol{L}$, we have found that the leptons are protected from obtaining GUT-scale masses via the inclusion of SU(3) adjoint superfields which, together with tritriplets $\boldsymbol{L}$, $\boldsymbol{Q}_\mathrm{L}$, and $\boldsymbol{Q}_\mathrm{R}$, are irreducible representations (irreps) of the E_8 symmetry group. This novel scenario is in the focus of our further discussion.

III. MINIMAL E_6-EXTENDED T-GUT MODEL

The proposed $[\mathbb{Z}_2 \times \mathbb{Z}_3]$-symmetric E_6-extended model, where the problem of SUSY T-GUT breaking is consistently resolved, preserves all the well-known attractive features of T-GUTs. The chiral superfield content of this model transforms as $(\mathbf{8},\mathbf{1})$, $(\mathbf{3},\mathbf{27})$, and $(\mathbf{1},\mathbf{78})$ of $\mathrm{SU}(3)_\mathrm{F} \times \mathrm{E}_6$, where $\mathrm{SU}(3)_\mathrm{F}$ is a global family symmetry. This set contains, in addition to the lepton and quark superfields $\boldsymbol{L}$, $\boldsymbol{Q}_\mathrm{L}$ and $\boldsymbol{Q}_\mathrm{R}$, chiral supermultiplets in the adjoint rep of $\mathrm{SU}(3)_\mathrm{A}$ (A=L, R, C, F) shown in Table I. The superpotential of this model reads

$$W = \sum_{\mathrm{A=L,R,C}} [\lambda_{\mathbf{78}} d_{abc} \Delta^a_\mathrm{A} \Delta^b_\mathrm{A} \Delta^c_\mathrm{A} + \mu_{\mathbf{78}} \Delta^a_\mathrm{A} \Delta^a_\mathrm{A}] + \lambda_{\mathbf{1}} d_{abc} \Delta^a_\mathrm{F} \Delta^b_\mathrm{F} \Delta^c_\mathrm{F} + \mu_{\mathbf{1}} \Delta^a_\mathrm{F} \Delta^a_\mathrm{F} + \lambda_{\mathbf{27}} \varepsilon_{ijk} \boldsymbol{Q}^i_\mathrm{L} \boldsymbol{Q}^j_\mathrm{R} \boldsymbol{L}^k, \qquad (2)$$

where $\lambda_{\mathbf{27}}$ is the unified quark-lepton Yukawa coupling, the subscript under the couplings denotes the E_6 irreps,

TABLE I. The minimal chiral superfield content of the SUSY $[\mathrm{SU}(3)]^3 \times \mathrm{SU}(3)_\mathrm{F} \subset \mathrm{E}_8$ model [with global family $\mathrm{SU}(3)_\mathrm{F}$].

Superfield	$\mathrm{SU}(3)_\mathrm{C}$	$\mathrm{SU}(3)_\mathrm{L}$	$\mathrm{SU}(3)_\mathrm{R}$	$\mathrm{SU}(3)_\mathrm{F}$
Lepton $(\boldsymbol{L}^i)^l_r$	1	$\mathbf{3}^l$	$\bar{\mathbf{3}}_r$	$\mathbf{3}^i$
Right quark $(\boldsymbol{Q}^i_\mathrm{R})^r_x$	$\bar{\mathbf{3}}_x$	$\mathbf{1}$	$\mathbf{3}^r$	$\mathbf{3}^i$
Left quark $(\boldsymbol{Q}^i_\mathrm{L})^x_l$	$\mathbf{3}^x$	$\bar{\mathbf{3}}_l$	$\mathbf{1}$	$\mathbf{3}^i$
Color adjoint Δ^a_C	$\mathbf{8}^a$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Left adjoint Δ^a_L	$\mathbf{1}$	$\mathbf{8}^a$	$\mathbf{1}$	$\mathbf{1}$
Right-adjoint Δ^a_R	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{8}^a$	$\mathbf{1}$
Family adjoint Δ^a_F	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{8}^a$

$d_{abc} \equiv 2\mathrm{Tr}[\{T_a, T_b\}T_c]$ are the totally symmetric SU(3) coefficients, $\boldsymbol{Q}^i_\mathrm{L}\boldsymbol{Q}^j_\mathrm{R}\boldsymbol{L}^k \equiv (\boldsymbol{Q}^i_\mathrm{L})^x_l(\boldsymbol{Q}^j_\mathrm{R})^r_x(\boldsymbol{L}^k)^l_r$, and summation over repeated indices is always implied. Furthermore, $\boldsymbol{L}$ unifies the light Higgs scalar and lepton sectors while $\boldsymbol{Q}_\mathrm{L}$ and $\boldsymbol{Q}_\mathrm{R}$ contain the SM quarks. In what follows, we refer to this model as the SUSY Higgs-unified trinification (or, shortly, SHUT) model.

The soft SUSY-breaking potential contains

$$\begin{aligned} V^\mathrm{G}_\mathrm{soft} = \{ & m^2_{\mathbf{27}}\tilde{L}\tilde{L}^\dagger + m^2_{\mathbf{78}}\tilde{\Delta}^{*a}_\mathrm{L}\tilde{\Delta}^a_\mathrm{L} + [b_{\mathbf{78}}\tilde{\Delta}^a_\mathrm{L}\tilde{\Delta}^a_\mathrm{L} \\ & + d_{abc}(A_{\mathbf{78}}\tilde{\Delta}^a_\mathrm{L}\tilde{\Delta}^b_\mathrm{L}\tilde{\Delta}^c_\mathrm{L} + C_{\mathbf{78}}\tilde{\Delta}^{*a}_\mathrm{L}\tilde{\Delta}^b_\mathrm{L}\tilde{\Delta}^c_\mathrm{L}) \\ & + A_G\tilde{\Delta}^a_\mathrm{L}T^a_\mathrm{L}(\tilde{L}^\dagger\tilde{L} + \tilde{Q}^\dagger_\mathrm{L}\tilde{Q}_\mathrm{L}) + \mathrm{c.c.}]\} \ltimes \mathbb{Z}_3 \\ & + [A_{\mathbf{27}}\varepsilon_{ijk}\tilde{Q}^i_\mathrm{L}\tilde{Q}^j_\mathrm{R}\tilde{L}^k + \mathrm{c.c.}] \end{aligned} \quad (3)$$

accounting for gauge adjoint scalars $\tilde{\Delta}^a_\mathrm{L,R,C}$, and

$$\begin{aligned} V^\mathrm{Gl}_\mathrm{soft} = {} & m^2_{\mathbf{1}}\tilde{\Delta}^{*a}_\mathrm{F}\tilde{\Delta}^a_\mathrm{F} + \{b_{\mathbf{1}}\tilde{\Delta}^a_\mathrm{F}\tilde{\Delta}^a_\mathrm{F} + A_{\mathbf{1}}d_{abc}\tilde{\Delta}^a_\mathrm{F}\tilde{\Delta}^b_\mathrm{F}\tilde{\Delta}^c_\mathrm{F} \\ & + A_F\tilde{\Delta}^a_\mathrm{F}(\tilde{L}^\dagger T^a_\mathrm{F}\tilde{L}) \ltimes \mathbb{Z}_3 + \mathrm{c.c.}\} \end{aligned} \quad (4)$$

for interactions involving family octets $\tilde{\Delta}^a_\mathrm{F}$, where T^a_A are the $\mathrm{SU}(3)_\mathrm{A}$ generators such that $\tilde{L}^\dagger T^a_\mathrm{L}\tilde{L} \equiv (\tilde{L}^*_k)^r_l(T^a_\mathrm{L})^l_{l'}(\tilde{L}^k)^{l'}_r$ etc., and summation over $\mathbb{Z}_3$ permutations is implied by the symbol $\ltimes\mathbb{Z}_3$. For completeness, we also include soft SUSY-breaking interactions in the fermion sector,

$$\mathcal{L}^\mathrm{ferm}_\mathrm{soft} = \left\{-\frac{1}{2}M_0\tilde{\lambda}^a_\mathrm{L}\tilde{\lambda}^a_\mathrm{L} - M'_0\tilde{\lambda}^a_\mathrm{L}\Delta^a_\mathrm{L} + \mathrm{H.c.}\right\} \ltimes \mathbb{Z}_3. \quad (5)$$

Here, besides the gaugino Majorana mass M_0, the symmetry allows a Dirac mass term parameterized by M'_0.

Notably, by setting all the soft SUSY-breaking parameters to zero the model still allows for the trinification SSB with a T-GUT-breaking but SUSY-preserving stable vacuum, giving rise to an effective SUSY LR-symmetric model below the GUT scale. At the moment, however, it is unclear if one could generate a consistent soft SUSY-breaking and gauge symmetry SSB in such an effective model providing a large splitting between the GUT and SM energy scales as required by phenomenology. We leave this open question to further studies taking into account the generic soft SUSY-breaking sector in the considered T-GUT as specified above.

In SUSY models with Dirac gauginos (such as minimal supersymmetric SM) the additional adjoint superfields spoil the gauge couplings' unification. This problem is resolved in the so-called minimal Dirac-gaugino supersymmetric standard model [45,46] inspired by $\mathrm{SU}(3)^3$ T-GUTs. In the SHUT model this problem is also resolved, but in a more elegant way, offering a framework that accommodates both the Dirac gauginos and the unified gauge coupling g_U. Furthermore, the proton is stabilized to all orders in perturbation theory due to an accidental $\mathrm{U}(1)_\mathrm{B}$. This global baryon symmetry is then preserved all the way down to the SM scale since none of the $(\tilde{Q}_\mathrm{L}, \tilde{Q}_\mathrm{R})$ squarks carrying the baryon number $(B = +1/3, -1/3)$ acquire a VEV [41] (see also Ref. [13]).

IV. SUSY T-GUT SYMMETRY BREAKING

The presence of family $\mathrm{SU}(3)_\mathrm{F}$ symmetry together with adjoint superfields $\boldsymbol{\Delta}^a_\mathrm{A}$ allows for a consistent trinification SSB which is rather clean compared to older SUSY T-GUT realizations. It also provides, in particular, SM-like fermion candidates whose masses are protected from GUT-scale contributions. Choosing a VEV along the $\tilde{\Delta}^8_\mathrm{A}$ direction yields the rank-preserving trinification SSB

$$\mathrm{SU}(3)_\mathrm{A} \to \mathrm{SU}(2)_\mathrm{A} \times \mathrm{U}(1)_\mathrm{A}, \qquad \mathrm{A} = \mathrm{L, R, F}. \quad (6)$$

Such a VEV choice is

$$\langle\tilde{\Delta}^8_\mathrm{L}\rangle \equiv v_\mathrm{L}, \qquad \langle\tilde{\Delta}^8_\mathrm{R}\rangle \equiv v_\mathrm{R}, \qquad \langle\tilde{\Delta}^8_\mathrm{F}\rangle \equiv v_\mathrm{F} \quad (7)$$

(where $v_\mathrm{L} = v_\mathrm{R} \equiv v$ is required by vacuum stability) which provides the SSB scheme

$$\begin{aligned} & [\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R}] \ltimes \mathbb{Z}_3 \times \mathrm{SU}(3)_\mathrm{F} \\ & \xrightarrow{v, v_\mathrm{F}} \mathrm{SU}(3)_\mathrm{C} \times [\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \\ & \quad \times \mathrm{U}(1)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{R}] \ltimes \mathbb{Z}_2 \times \mathrm{SU}(2)_\mathrm{F} \times \mathrm{U}(1)_\mathrm{F}, \end{aligned} \quad (8)$$

in addition to implicit accidental symmetries such as $\mathrm{U}(1)_\mathrm{B}$. Here, the square brackets denote parts gathered under the permutation symmetries.

After the T-GUT symmetry breaking (8) the fermionic tritriplets L, Q_L, and Q_R are split into blocks revealing, e.g., massless $\mathrm{SU}(2)_\mathrm{L}$ $[\mathrm{SU}(2)_\mathrm{R}]$ doublets of leptons $E_\mathrm{L} \equiv (e_\mathrm{L}, \nu_\mathrm{L})$ $[E_\mathrm{R} \equiv (e^c_\mathrm{R}, \nu^c_\mathrm{R})]$ and quarks $q_\mathrm{L} \equiv (u_\mathrm{L}, d_\mathrm{L})$ $[q_\mathrm{R} \equiv (u^c_\mathrm{R}, d^c_\mathrm{R})]$, whose first and second generations form $\mathrm{SU}(2)_\mathrm{F}$ doublets. Notably, the matching of Yukawa couplings in subsequent EFT scenarios is greatly simplified due to the unified Yukawa interactions in the considered T-GUT.

V. LEFT-RIGHT-SYMMETRIC EFFECTIVE THEORY

We have found that the high-scale SHUT model gives rise to a non-SUSY LR $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}}$-symmetric EFT [Eq. (8)] as long as the quadratic and trilinear soft SUSY-breaking terms are small compared to the GUT scale. Here, we briefly discuss an important class of its characteristic low-energy scenarios where (i) all adjoint scalars $\tilde{\Delta}_{\mathrm{L,R,C}}$ as well as $\tilde{\Delta}_{\mathrm{F}}^{1,2,3,8}$ are heavy, thus are integrated out at the T-GUT-breaking (or, simply, GUT) scale, and (ii) the fundamental scalars $\tilde{L}$ are lighter than the GUT scale and are kept in the LR-symmetric EFT. This is indeed the most natural choice as the masses of the latter are solely governed by soft SUSY-breaking interactions while those of the former also contain large $\mathcal{F}$- and $\mathcal{D}$-term contributions of the order of the GUT scale. In particular, assuming for simplicity the superpotential and soft SUSY-breaking parameters to be real, it follows from Eqs. (3) and (4) that the masses of the scalar components of the tritriplets $\boldsymbol{L}$, $\boldsymbol{Q}_{\mathrm{L}}$, and $\boldsymbol{Q}_{\mathrm{R}}$ are of the form

$$m^2_{\tilde{\varphi}_i} = m^2_{\mathbf{27}} + c^i_1 A_{\mathrm{G}} v + c^i_2 A_{\mathrm{F}} v_{\mathrm{F}}, \qquad (9)$$

where the index i runs over all fundamental scalars and $c^i_{1,2}$ are irrational constants. We can now relate all dimensionful parameters to the T-GUT-breaking VEV as $m^2_{\mathbf{27}} \equiv \alpha_{\mathbf{27}} v^2$, $A_{\mathrm{G}} \equiv \sigma_{\mathrm{G}} v$, $A_{\mathrm{F}} \equiv \sigma_{\mathrm{F}} v$, and $v_{\mathrm{F}} \equiv \beta v$. Here, $\alpha_{\mathbf{27}}, \sigma_{\mathrm{G}}, \sigma_{\mathrm{F}} \ll 1$ are small, as they parametrize unknown details of soft SUSY breaking, while $\beta \sim \mathcal{O}(1)$ such that both gauge and family SSBs occur simultaneously. This allows us to recast the scalar masses as

$$m^2_{\tilde{\varphi}_i} = v^2(\alpha_{\mathbf{27}} + c^i_1 \sigma_{\mathrm{G}} + c^i_2 \beta \sigma_{\mathrm{F}}) \equiv v^2 \omega_{\tilde{\varphi}_i}, \quad \omega_{\tilde{\varphi}_i} \ll 1. \qquad (10)$$

Interestingly, the light scalar spectrum of the effective LR-symmetric model is fully determined by three independent small parameters characterizing the soft SUSY-breaking sector and thus is protected from gaining the GUT-scale radiative corrections. Choosing, for example, $\omega_{\tilde{H}^{(3)}} \equiv \xi$, $\omega_{\tilde{E}^{(1,2)}_{\mathrm{L,R}}} \equiv \delta$ and $\omega_{\tilde{H}^{(1,2)}} \equiv \kappa$, one obtains

$$\begin{aligned}
m^2_{\tilde{H}^{(3)}} &= v^2 \xi, & m^2_{\tilde{H}^{(1,2)}} &= v^2 \kappa, \\
m^2_{\tilde{E}^{(3)}_{\mathrm{L,R}}} &= v^2(\delta + \xi - \kappa), & m^2_{\tilde{E}^{(1,2)}_{\mathrm{L,R}}} &= v^2 \delta, \\
m^2_{\tilde{\phi}^{(3)}} &= v^2(2\delta + \xi - 2\kappa), & m^2_{\tilde{\phi}^{(1,2)}} &= v^2(2\delta - \kappa), \\
m^2_{\tilde{q}^{(3)}_{\mathrm{L,R}}} &= \frac{1}{3} v^2(\delta + 3\xi - \kappa), & m^2_{\tilde{q}^{(1,2)}_{\mathrm{L,R}}} &= \frac{1}{3} v^2(\delta + 2\kappa), \\
m^2_{\tilde{D}^{(3)}_{\mathrm{L,R}}} &= \frac{1}{3} v^2(4\delta + 3\xi - 4\kappa), & m^2_{\tilde{D}^{(1,2)}_{\mathrm{L,R}}} &= \frac{1}{3} v^2(4\delta - \kappa),
\end{aligned} \qquad (11)$$

where ξ, δ and κ determine all possible mass hierarchies in the scalar spectrum in the LR-symmetric EFT at the GUT scale. Together with quartic, Yukawa, and gauge couplings, they control the initial conditions and shape of the RG flow and therefore define a particular SSB scheme affecting the features of the low-energy EFT limit. For example, setting $\kappa \ll \xi \ll \delta$ one finds that $m^2_{\tilde{H}^{(1,2)}} \ll m^2_{\tilde{H}^{(3)}} \ll m^2_{\mathrm{others}} \ll v^2$. One of the possible symmetry-breaking schemes down to the SM gauge group consists of two subsequent steps that can be induced by the VEVs $\langle \tilde{\phi}^{(3)} \rangle \equiv \langle (\tilde{L}^3)^3_3 \rangle$ and $\langle \tilde{\nu}^{(2)}_{\mathrm{R}} \rangle \equiv \langle (\tilde{L}^2)^3_1 \rangle$ at well-separated scales. This is represented by the following SSB chain:

$$\begin{aligned}
&\mathrm{SU}(3)_{\mathrm{C}} \times [\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}} \times \mathrm{U}(1)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{R}}] \ltimes \mathbb{Z}_2 \\
&\xrightarrow{\langle \tilde{\phi}^{(3)} \rangle} \mathrm{SU}(3)_{\mathrm{C}} \times [\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}}] \ltimes \mathbb{Z}_2 \times \mathrm{U}(1)_{\mathrm{L+R}} \\
&\xrightarrow{\langle \tilde{\nu}^{(2)}_{\mathrm{R}} \rangle} \mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}},
\end{aligned} \qquad (12)$$

where only the gauge symmetry and $\mathbb{Z}_2$ are shown.

Consider the SSB chain (12) in more detail. Due to the presence of both Majorana and Dirac mass terms in the fermion-adjoint sector, with a large splitting one recovers light neutralino- and gluinolike states in the LR-symmetric EFT with masses $m_{\mathcal{S}_{\mathrm{L,R}}} \simeq m_{\mathcal{T}_{\mathrm{L,R}}} \simeq m_{\tilde{g}} \simeq 2M_0$ in terms of the soft SUSY-breaking parameter $M_0 \ll v \sim \mu_{\mathbf{78}}$. Here, the $\mathrm{SU}(2)_{\mathrm{L,R}}$ triplet $\mathcal{T}_{\mathrm{L,R}}$ and singlet $\mathcal{S}_{\mathrm{L,R}}$ states emerge from a decomposition of the $\mathrm{SU}(3)_{\mathrm{L,R}}$ octets as $\mathbf{8} \to \mathbf{3}_0 \oplus \mathbf{2}_1 \oplus \mathbf{2}_{-1} \oplus \mathbf{1}_0$, and $\tilde{g}$ is the lightest gluino. On the other hand, as long as $M_0 \sim \langle \tilde{\phi}^{(3)} \rangle \ll v$, these gauginolike states will be integrated out at the $\mathcal{O}(\langle \tilde{\phi}^{(3)} \rangle)$ scale. Thus, in the resulting $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}} \times \mathrm{U}(1)_{\mathrm{L+R}}$ EFT, the gaugino-lepton mass terms do not appear and the SM fermions are guaranteed to remain massless until the electroweak scale. Conveniently, the charges of the weak-singlet (non-SM) down-type quarks allow them to gain masses at the LR-breaking scales $\langle \tilde{\phi}^{(3)} \rangle$, $\langle \tilde{\nu}^{(2)}_{\mathrm{R}} \rangle$ via the high-scale Yukawa terms of the form $Q_{\mathrm{L}} Q_{\mathrm{R}} \tilde{L}$.

VI. SIGNIFICANCE, EXPECTATIONS AND FUTURE WORK

The proposed E_6-extended SHUT model represents a promising way of unifying the light Higgs scalar and SM lepton sectors into the same supermultiplet $\boldsymbol{L}$, where [due to the trinification SSB via adjoint scalar VEVs and the family $\mathrm{SU}(3)_{\mathrm{F}}$] the SM fermions are protected from gaining masses in the high-scale model, in consistency with the SM. The inclusion of $\mathrm{SU}(3)_{\mathrm{F}}$ also results in the high-scale unification of the tree-level quark-lepton Yukawa couplings in the current framework [see $\lambda_{\mathbf{27}}$ in Eq. (2)]. Due to the emergent Yukawa and Higgs-lepton unification properties, the SHUT model has a relatively low number of free parameters at the GUT scale without introducing additional Higgs multiplets besides those in E_8 and also without assuming any universality in the soft SUSY-breaking sector. While potentially sharing some of the key features of the non-SUSY T-GUT scenario discussed in Ref. [41], the SHUT model brings a straightforward explanation to some of its seemingly arbitrary characteristics such as the presence of scalars and fermions with the same quantum numbers.

In particular, in Ref. [41] it was demonstrated that in the non-SUSY T-GUT the LR symmetry breaking down to the SM gauge group can be initiated radiatively through the RG evolution. The circumstances under which the model leads to a realistic mass spectrum at lower energies were also explored, as well as aspects of its one-loop stability. Indeed, due to the running of a mass squared of a scalar $\mathrm{SU}(2)_{\mathrm{R/F}}$ bidoublet $(\tilde{e}_{i=1,2}, \tilde{\nu}_{i=1,2})$ to a negative value at lower scales, the SSB can be triggered in the LR-symmetric EFT with a residual global $\mathrm{SU}(2)_{\mathrm{F}}$ down to the SM gauge symmetry [cf. the last SSB step in Eq. (12)]. Similar low-energy features could be present in the considered SHUT model as a plausible possibility, though they are not immediately guaranteed since its mass spectra differ from that of Ref. [41]. A better understanding of the radiative symmetry breaking in the resulting LR-symmetric EFT which determines the structure of the SM-like theory at low energies should be the subject of future studies.

In the SM-like EFT, resulting from the chain (12), the three lightest SM Higgs $\mathrm{SU}(2)_{\mathrm{L}}$ doublets originating from the scalar $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}} \times \mathrm{SU}(2)_{\mathrm{F}}$ tridoublet in the LR-symmetric EFT are expected to develop VEVs breaking the electroweak symmetry. As long as this property holds true, it provides a correct mass scale for the SM quarks in the second and third generations as well as gives rise to the Cabbibo mixing pattern at tree level. While there are no tree-level Higgsino, SM lepton, and first-generation quark masses in the high-scale theory, those can, in principle, be regenerated radiatively as soon as the LR and electroweak symmetries are broken. The EFT fermion mass spectra should thus be explored at least to one-loop order in following studies.

VII. CONCLUSIONS

By unifying light Higgs bosons and SM leptons in the same supermultiplet of trinification, by breaking the trinification symmetry with adjoint scalar VEVs, and by introducing a global family symmetry, the SHUT model protects the SM fermions from gaining masses until the electroweak symmetry is broken while still ensuring the proton stability. The apparent simplicity of the SHUT model, originating from its gauge, Yukawa, and Higgs-lepton unification at the trinification breaking scale, makes it a very interesting candidate for further theoretical and phenomenological studies. Depending on the chosen symmetry-breaking scheme as well as on values of the high-scale couplings and the hierarchy between them, the path down to an effective SM-like theory could lead to vast and yet unexplored low-energy phenomena. While those are yet to be understood in full detail, the SHUT model presented here shows potential for reviving the trinification GUT model building.

The first immediate task in further developments of the proposed high-scale SHUT model is to derive the basic properties of its SM-like EFT limit (at least, to one loop) and then to search for possible deviations from the characteristic SM signatures. This would allow us to set constraints on the SHUT parameter space and, possibly, to predict new smoking gun signals of new physics specific to the corresponding LR-symmetric EFT. The latter would then offer a plethora of opportunities for phenomenological studies of potentially observable beyond-SM phenomena in connection with the ongoing LHC and astroparticle physics searches.

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