

Loop neutrino masses from $d = 7$ operator

R. Cepedello,^a M. Hirsch^a and J.C. Helo^{b,c}

^a*AHEP Group, Instituto de Física Corpuscular – C.S.I.C./Universitat de València, Edificio de Institutos de Paterna, Apartado 22085, E-46071 València, Spain*

^b*Departamento de Física, Facultad de Ciencias, Universidad de La Serena, Avenida Cisternas 1200, La Serena, Chile*

^c*Centro-Científico-Tecnológico de Valparaíso, Casilla 110-V, Valparaíso, Chile*

E-mail: ricepe@ific.uv.es, mahirsch@ific.uv.es, juancarlos.helo@usm.cl

ABSTRACT: We discuss the generation of small neutrino masses from $d = 7$ 1-loop diagrams. We first systematically analyze all possible $d = 7$ 1-loop topologies. There is a total of 48 topologies, but only 8 of these can lead to “genuine” $d = 7$ neutrino masses. Here, we define genuine models to be models in which neither $d = 5$ nor $d = 7$ tree-level masses nor a $d = 5$ 1-loop mass appear, such that the $d = 7$ 1-loop is the leading order contribution to the neutrino masses. All genuine models can then be organized w.r.t. their particle content. We find there is only one diagram with no representation larger than triplet, while there are 22 diagrams with quadruplets. We briefly discuss three minimal example models of this kind.

KEYWORDS: Beyond Standard Model, Neutrino Physics

ARXIV EPRINT: [1705.01489](https://arxiv.org/abs/1705.01489)

Contents

1	Introduction	1
2	Preliminaries	3
2.1	Tree-level $d = 5$	3
2.2	Tree-level $d = 7$	4
2.3	1-loop $d = 5$	4
3	Classification	6
3.1	Topologies	6
3.2	Diagrams: minimal $SU(2)_L$ representations	8
3.3	Example models	10
3.3.1	Triplet model	10
3.3.2	Quadruplet model	11
3.3.3	Quintuplet model	12
4	Summary	13
A	Topologies and genuine diagrams	14
A.1	Topologies	14
A.2	Genuine diagrams	16

1 Introduction

If neutrinos are Majorana particles, one can roughly estimate their mass as:

$$m_\nu \propto \frac{v^2}{\Lambda} \times \left(\frac{1}{16\pi^2}\right)^n \times \epsilon \times \left(\frac{v}{\Lambda}\right)^{d-5}. \quad (1.1)$$

Here, Λ is the energy scale of new physics, where lepton number violation (LNV) occurs and v is the standard model vacuum expectation value. The different terms in eq. (1.1) can be understood easily. The first term corresponds to the famous Weinberg operator, $\mathcal{O}^W \equiv \mathcal{O}^{d=5} \propto LLHH$ [1]. This operator can be generated either at tree-level or at loop level. The second term in eq. (1.1) takes into account this fact, with $n = 0, 1, 2, \dots$ being the number of loops. Then, there are models of neutrino mass, in which the Weinberg operator is suppressed by some small factor ϵ , which could be either due to some small coupling in the corresponding model or due to some nearly conserved symmetry. R-parity violating supersymmetry is an example of the former [2, 3], models such as the inverse [4] or the linear [5, 6] seesaw are examples of the latter. And, finally, neutrino masses could be due to higher dimensional operators. This is expressed by the last term in eq. (1.1), with $d = 5, 7, \dots$ the dimension of the operator.

In this paper we will focus on $d = 7$ neutrino mass models at 1-loop order. Our aim is to give a systematic analysis of such models, constructing first all possible 1-loop topologies and then identify those topologies, which *allow to construct genuine* models. Here, we define *genuine* models such models, where the 1-loop $d = 7$ contribution to the neutrino mass gives the leading order contribution. This assumption implies, of course, that both the $d = 5$ and the $d = 7$ tree-level, as well as the $d = 5$ 1-loop, contributions should be absent.

$d = 5$ neutrino masses have been studied extensively in the literature. In [7] it was shown that there are only three tree-level realizations of \mathcal{O}^W at tree-level. A systematic analysis of the Weinberg operator at 1-loop level was presented in [8], at 2-loop level in [9], see also [10] for a general discussion of tree versus loop neutrino masses.

Disregarding derivative operators, the authors of [11] have written down all $\Delta L = 2$ operators up to $d = 11$. Only one of these operators is important for us here:

$$\mathcal{O}^{d=7} \propto LLHHHH^\dagger \tag{1.2}$$

All other $d = 7$ operators in the list of [11] will lead to $d = 5$ 1-loop neutrino mass models, while the $d = 9$ and $d = 11$ operators can lead only to $d = 7$ neutrino masses, if the underlying model is 2-loop or higher. Note that $\Delta L = 2$ operators with derivatives have been studied in [12]. Two operators with derivatives at $d = 7$ exist, but neither can lead to a 1-loop $d = 7$ model, see also the discussion in [13].

Bonnet et al. [14] analyzed the $d = 7$ operator of eq. (1.2) at tree-level in detail. As noted in this work, the $d = 7$ operator of eq. (1.2) will always also generate a higher order $d = 5$ neutrino mass:

$$\frac{1}{\Lambda^3} LLHHHH^\dagger \rightarrow \frac{1}{16\pi^2} \frac{1}{\Lambda} LLHH \tag{1.3}$$

One can straightforwardly estimate that this loop contribution will become more important than the tree-level one if $(\Lambda/v) \gtrsim 4\pi$. This means $\Lambda \lesssim 2 \text{ TeV}$ is required for the $d = 7$ contribution to dominate. Since this is unavoidable in the standard model (SM), the authors of [14] considered a two Higgs doublet extension of the SM in their discussion of the $d = 7$ tree-level neutrino mass.¹ We instead will stick to only the SM Higgs and take eq. (1.3) as a motivation that any $d = 7$ model of neutrino mass *must have new particles below 2 TeV*, otherwise it will not give the leading contribution to the neutrino mass matrix.

As mentioned above, both $d = 5$ and $d = 7$ tree-level contributions should be forbidden, otherwise the $d = 7$ 1-loop contribution might be just some minor correction to the neutrino mass matrix. Absence of these lower order contributions could be attributed to either: (i) the existence of some symmetry; or (ii) absence of fields which generate neutrino masses at lower order. An example of the former at $d = 5$ is the scotogenic model [15]. In this model, a right-handed neutrino (plus an extra doublet scalar) exists, but due to a Z_2 symmetry, there is no tree-level $d = 5$ neutrino mass.² Instead neutrinos have mass at ($d = 5$) 1-loop

¹ HH^\dagger is a singlet under any discrete symmetry. With more than one Higgs it is possible to introduce an additional discrete symmetry, under which the two Higgses transform differently.

²The well-known bonus of the Z_2 symmetry is that it allows to “stabilize” the lightest Z_2 odd particle, thus relating the stability of the dark matter to the generation of neutrino masses.

order. The classic example for (ii) is the Zee model [16]. In the Zee model, none of the particles necessary for a tree-level seesaw exist. Instead an additional charged singlet scalar (plus an additional doublet scalar) generate neutrino masses radiatively.

Two comments might be in order. First, with the use of non-Abelian discrete symmetries it is possible to construct viable models, genuine in the sense that they can give the leading contribution to the neutrino mass matrix. We will not discuss in detail such models, since the use of discrete symmetries for $d = 7$ neutrino masses have been discussed in this context already in a number of references, see for example [14, 17–20].

Second, the example models, which we will discuss later on, all have explicit lepton number violation (LNV). One could construct extensions of these models, in which the LNV is spontaneous. In that case, a massless Goldstone boson would appear, usually called Majoron in the literature. We will not discuss the phenomenology of Majorons here and only note in passing that no new topologies would be generated in such models, with respect to the explicitly LNV models we consider.

Finally, we mention that we will also not consider $d = 7$ operators with additional singlets, see for example [21, 22].

The rest of this paper is organized as follows. In the next section, we will first give a short summary of neutrino masses at tree-level at $d = 5$ and $d = 7$, as well as $d = 5$ at 1-loop order. This provides the basis for the discussion in section 3. Section 3 then provides the core of our present work. It discusses all possible topologies and classifies them into different groups. The section then also introduces the three most minimal example models that one can construct at $d = 7$ 1-loop order. We then close with a short summary. More complete lists of topologies and diagrams are relegated to the appendix.

2 Preliminaries

Since we are interested in identifying models, in which a 1-loop $d = 7$ diagram gives the leading order contribution to the neutrino mass matrix, we first need to discuss briefly neutrino masses at lower order. When discussing possible models will use S (and ϕ) for scalars and ψ (χ) for fermions. For a more compact notation we will also use a notation which gives the $SU(2)_L$ representation and hypercharge in the form \mathbf{R}_Y with a superscript S or F , where necessary, i.e. for example $\mathbf{5}_1^S$ is a scalar **5-plet** with $Y = 1$. Note that, for some of the fields, particular symbols are common in the literature, such as ν_R , Δ and Σ for the type-I, type-II and type-III seesaw.

2.1 Tree-level $d = 5$

As noted in [7], there are only three possibilities to de-construct the Weinberg operator at tree-level. A seesaw type-I is generated via the introduction of a right-handed neutrino, $\nu_R \equiv \mathbf{1}_0^F$ [23–25]. It generates Dirac mass terms for the active neutrinos via $\bar{\nu}_R H L$. For the singlet a Majorana mass term is allowed, $M_M \bar{\nu}_R^c \nu_R$, which implies $\Delta L = 2$. The type-II seesaw requires a scalar triplet $\Delta \equiv \mathbf{3}_1^S$ [26–28]. Here, the simultaneous presence of the couplings $L\Delta L$ and $H\Delta^\dagger H$ violates lepton number $\Delta L = 2$. And, finally, a type-III seesaw [29] can be generated with a fermionic triplet $\Sigma \equiv \mathbf{3}_0^F$. Here, the vector-like mass

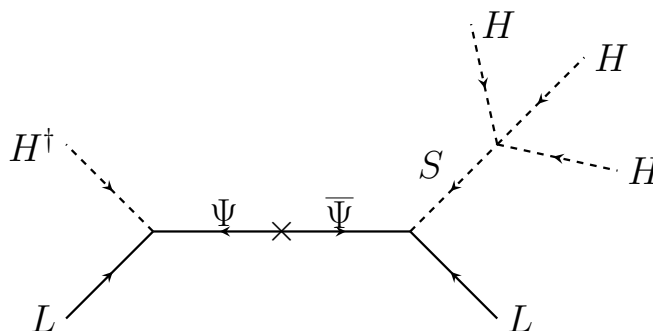


Figure 1. Tree-level $d = 7$ neutrino mass diagram, for the model described in [30]. $\Psi = \mathbf{3}_1^F$ and $S = \mathbf{4}_{3/2}^S$.

term $M_\Sigma \bar{\Sigma}^c \Sigma$ is the source of the lepton number violation. Note, that type-I and type-III seesaw are generated by the same topology, reducing the total number of $d = 5$ topologies at tree-level to two.

2.2 Tree-level $d = 7$

At $d = 7$ level one can construct five different topologies, one of which however can not lead to any renormalizable model. The remaining four topologies have been discussed in [14]. Only one of these topologies can generate a genuine $d = 7$ neutrino mass model in our sense, see figure 1. All other models will require additional symmetries to avoid $d = 5$ tree-level neutrino masses.

This genuine $d = 7$ tree-level model, BNT model in the following, was first discussed in [30]. For a discussion of lepton flavour violation in the BNT model see [31]. The model requires two new particles beyond the SM field content: (i) A (vector-like) triplet fermion, $\Psi = \mathbf{3}_1^F$.³ And (ii) a scalar quadruplet $S \equiv \mathbf{4}_{3/2}^S$. Note that the quadruplet is the smallest representation which allows a contraction $S^0(H^0)^3$.

2.3 1-loop $d = 5$

In addition to the tree-level, there are many 1-loop $d = 5$ models. The classical example is the Zee model [16]. A systematic analysis of all 1-loop $d = 5$ topologies has been given in [8]. In total, 6 topologies were found, but only two of them (denoted as T-1 and T-3) can give genuine models in our sense. All other topologies lead either to non-renormalizable models or diagrams with infinite loop integrals (thus representing loop corrections to tree-level quantities)⁴ or can be understood as finite 1-loop realizations of some particular vertex of one of the tree-level $d = 5$ seesaws. Topologies T-1 and T-3 lead to a total of four diagrams shown in figure 2. The Zee model [16] falls within category T-1-ii, the scotogenic model of Ma [15] is an example of T-3.

³Its vector partner $\mathbf{3}_{-1}^F$ is needed for a (effectively lepton number violating) mass term.

⁴Topology T-4 has two divergent and two finite diagrams. In [32] the diagram T-4-2-ii was used to generate a coupling $L\Delta L$ at one-loop. This diagram is classified as divergent in [8]. However, in [32] two diagrams of this type appear, with the infinity cancelled between diagrams. This can not be justified in terms of symmetry, instead it is due to the fact that lepton number is broken *softly* in the model of [32].

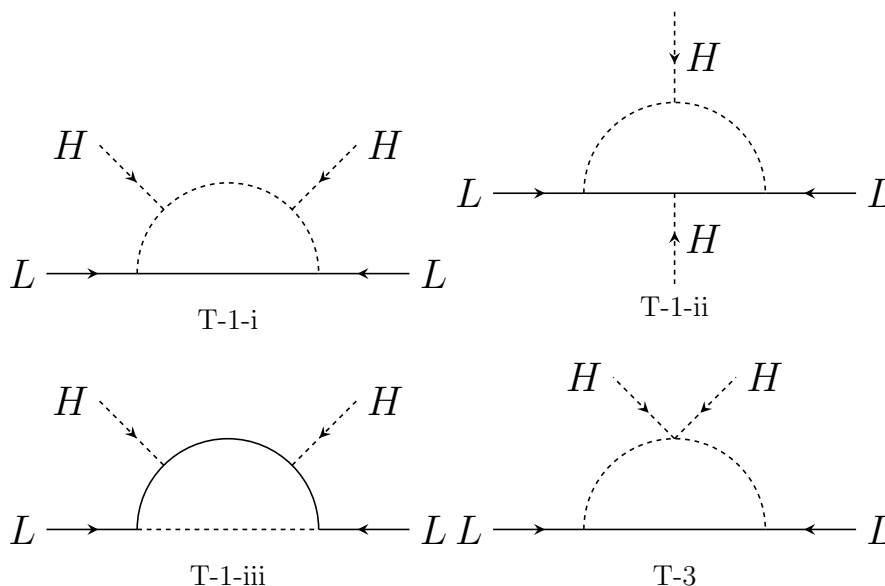


Figure 2. The four genuine $d = 5$ 1-loop neutrino mass diagrams in the notation of [8].

While at tree-level the size of the representations as well as the hypercharge of the new fields is fixed, at loop level there always exists a “tower” of possible models. This is easily understood. Consider, for example, the diagram T-3. The outside leptons couple to a scalar and a fermion. Since L is a $SU(2)$ doublet, the representation of the scalar and the fermion can be: $\mathbf{1} \otimes \mathbf{2}$, $\mathbf{2} \otimes \mathbf{3}$, $\mathbf{3} \otimes \mathbf{4}$, etc. Similarly at the four scalar vertex the smallest possibility is $\mathbf{2} \otimes \mathbf{2}$, but larger representations can be inserted, with the only constraint that the product of the two scalars can build a triplet. In the same way, the hypercharge of the internal particles is fixed only up to an additive constant x , that runs in the loop.⁵

The minimal possibility to build a model for T-3 is then that the fermion is a $\nu_R = \mathbf{1}_0^F$, while the scalar is a doublet $\mathbf{2}_{1/2}^S$, i.e. the well-known scotogenic model. This is minimal in the sense that it uses the smallest representations and the smallest value of the hypercharge possible, i.e. $x = 0$. However, $\mathbf{2}_{1/2}^S$ can not be the SM Higgs, it must be an additional inert doublet.

Note that in our sense, strictly speaking, the scotogenic model is not a “genuine” model, since it requires an additional symmetry (in the minimal case a simple Z_2) to avoid the tree-level $d = 5$ type-I seesaw. This does not mean, however, that topology T-3 is non-genuine: this topology has a finite loop integral and thus, no tree-level counter term is needed in models that generate this topology. Rather, in order to avoid the type-I seesaw contribution without the use of an additional symmetry, requires us to introduce fields with larger hypercharges. The smallest possible choice is: $\mathbf{1}_1^F$ (together with its vector partner) and two scalars $\mathbf{2}_{1/2}^S$ and $\mathbf{2}_{3/2}^S$.

⁵Of course, not all choices of x will lead to phenomenologically acceptable models.

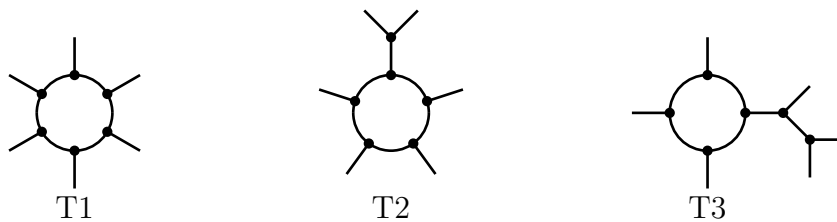


Figure 3. Generating topologies, starting from the simplest topology, containing only 3-point vertices with all 6 particles connected to the loop, T1. Subsequent topologies are found by removing systematically particles attached to the loop and reconnecting them to outside particles, as shown for the examples T2 and T3. See text.

3 Classification

In this section, we will discuss the classification of the different $d = 7$ 1-loop diagrams. We first construct all possible 1-loop topologies with six external legs and then discard in different steps those topologies that can not lead to genuine models. For the remaining topologies we order all possible diagrams into different classes, depending on the minimum size of the largest required $SU(2)_L$ representation appearing in the corresponding diagram.

We note in passing that we will not discuss colour in detail, because colour assignments can be trivially added: all particles outside loops must be necessarily colour singlets, while pairs of particles in loops can always be assigned colour in combinations $\mathbf{X} + \bar{\mathbf{X}}$, for $\mathbf{X} = \mathbf{1}, \mathbf{3}, \dots$, which then couple to “outside” colour singlet particles.

3.1 Topologies

We construct all possible 1-loop topologies with six external legs, discarding from the start all self-energy corrections. This construction can be done in different ways and we used two different procedures to assure that all possible topologies were found. In total there are 48 possible topologies. The complete list is shown in the appendix.

We will briefly describe the methods we used to find the topologies. The first procedure consists in taking the five $d = 7$ tree-level topologies and to generate loops in all possible combinations, connecting either lines to lines or lines to vertices or vertices to vertices, using only 3-point and 4-point vertices. From this list one has to discard in the end all duplicates.

The second procedure starts from the most simple realization of a 1-loop topology adding six external lines to the loop using only 3-point vertices, as shown in figure 3. From this topology, all other topologies can be found by systematically removing lines attached to the loop and adding them to an outside particle generating a new 3-point vertex, as shown in the figure for the examples of T2, T3 etc. Once all possible topologies with only 3-point vertices are found, all remaining topologies can be generated from the earlier ones by shrinking one line connecting two 3-point vertices to one new 4-point vertex, see the example figure 4. Again, this procedure produces duplicates, which have to be identified and discarded.

This second procedure provides a systematic construction of all possible topologies in a more intuitive way than the first one described above. It allows to classify topologies

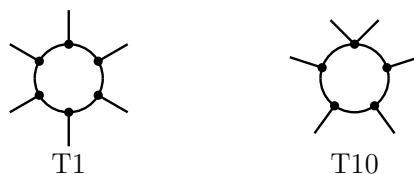


Figure 4. Constructing topologies with 4-point vertices from existing topologies with only 3-point vertices by “shrinking” one connecting line. Here shown for the example how T1 generates T10. See also text.

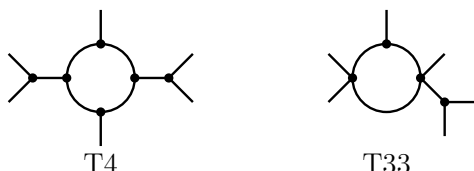


Figure 5. Two example $d = 7$ topologies which always will be accompanied by a tree-level seesaw $d = 5$ contributions to the neutrino mass matrix. For discussion see text.

according to the number of lines entering the loop, creating subgroups with the same number of 4-leg vertices. This procedure is the one we use for ordering the complete list of topologies, given in the appendix.

We then proceed to order topologies into different groups. We can discard immediately the six topologies shown in figure 17, because none of them can lead to a renormalizable model. The next step is to generate all possible diagrams and check if any field which generates neutrino masses at lower order is required.

Two examples of topologies, which always necessarily will be accompanied by a tree-level $d = 5$ seesaw, are shown in figure 5. These diagrams can be easily understood. Every topology with at least two 3-leg vertices on two external lines will always generate a vertex $LH\bar{\nu}_R$ (or $LH\bar{\Sigma}$ or $H\Delta^\dagger H$) and, thus a seesaw at tree-level, as in the example T4 figure 5 on the left. Topologies which contain one 3-leg vertex with two external lines isolated by a 4-leg vertex will always have a coupling of the type $L\Delta L$, as for example the topology T33 in figure 5, to the right. The 27 topologies, for which all diagrams can be excluded due to this argument, are given in figure 14. The topologies T7, T22, T23 and T24 in this figure are somewhat particular examples. For these a Δ always has to exist. One might think to bypass the Δ and introduce a quintuplet instead. However, the coupling of two doublets to a 5-plet is zero due to $SU(2)_L$.

Before moving on a brief comment might be in order. Integrals for the diagrams in topologies T4 and T33 are finite. One might therefore wonder, whether it is possible to forbid one of the “ingredients” of the tree-level $d = 5$ seesaw, say one particular vertex, via a discrete symmetry, only to generate it at 1-loop order. This was discussed at length for $d = 5$ 1-loop diagrams in [8]. At the $d = 7$ level, however, this will not be possible, since $H^\dagger H$ is a singlet under any discrete symmetry.

Next we turn to identifying topologies which generate diagrams reducible to 1-loop $d = 5$ models. This is not as straightforward as the tree-level case. In particular, in this



Figure 6. The particular piece of diagram that generates the 1-loop $d = 5$ diagram T-3, see figure 2. If this structure exists in any diagram, the vertex with the two scalars η, η' and two Higgses also always exist. This structure appears in many diagrams of the $d = 7$ topologies.

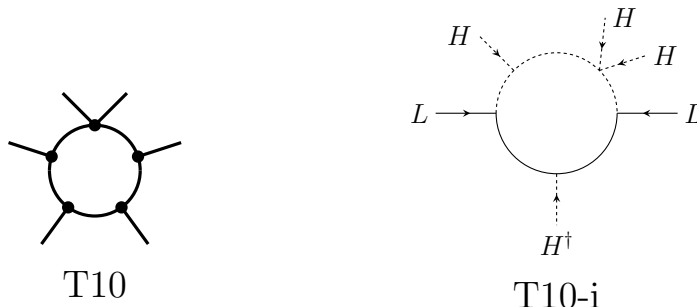


Figure 7. Topology T10 (left) and the only diagram derived from this topology, that can give a genuine model (right).

class of topologies many diagrams lead to $d = 5$ tree-level models, while only the remaining diagrams can lead to $d = 5$ 1-loop models. However, when the topology is highly symmetric, as for example in $T1$, one can always find a coupling between two internal fields and an external field which bypasses the H^\dagger , giving one of the diagrams of figure 2. In addition, any diagram containing the structure given in figure 6 can be reduced to the well-known diagram T-3 in figure 2. We list all topologies excluded due to these arguments in figure 15.

Then, there are diagrams which always contain the fields $\mathbf{4}_{3/2}^S$ and $\mathbf{3}_1^F$, responsible of generating neutrino mass at tree-level $d = 7$ [30]. Examples are diagrams of the topologies $T25, T29$ and $T35$ (figure 16), which are excluded as genuine ones due to this reason.

Finally, in the remaining 8 topologies that are not completely excluded by one of the above arguments, many but not all the diagrams do not lead to genuine models. For instance, from the 10 different diagrams that one can generate from topology $T10$ (figure 7 left), only one is not reducible to 1-loop $d = 5$ (figure 7 right).

In summary, from the initial 48 topologies only 8 have at least one genuine 1-loop $d = 7$ diagram. The excluded topologies are listed in the appendix in figures 14–17. The next step is to classify the surviving topologies in terms of the minimal $SU(2)_L$ representation needed to realize a genuine model.

3.2 Diagrams: minimal $SU(2)_L$ representations

In all $d = 7$ diagrams, in order to avoid neutrino masses at lower order, a minimal size for the $SU(2)_L$ representations of the model is required. We order the possible models according to the largest representation present in a given model. The “smallest” or minimal model that one can construct is then a model in which no representation larger than $SU(2)_L$ triplets is needed. The next smallest possibility is models with quadruplets. Here, one can distinguish

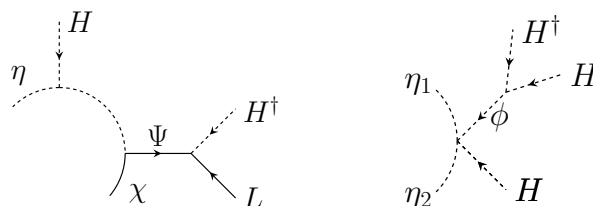


Figure 8. Structures that appear in topologies T12 (left) and T13 (right) which require at least a quadruplet and a singlet running inside the loop to avoid lower order contributions. For discussion see text.

three different subgroups: (i) diagrams in which one quadruplet is needed inside the loop; (ii) diagrams in which one quadruplet appears outside the loop and internal particles need not be larger than triplets; and (iii) models in which at least two quadruplets are needed. We will discuss these three possibilities in reverse order and then proceed to briefly discuss the triplet diagram.

For external fields, finding the minimal representation is straightforward. A recurrent example in most of the diagrams of the appendix is that of the vertex HH^\dagger -scalar. Since $2 \otimes 2 = 3 + 1$, the scalar could be a trivial singlet or the triplet $\phi \equiv \mathbf{3}_0^S$. The former case is directly reducible to a $d = 5$ diagram, the latter is the one we are interested in. The same principle applies to the diagrams given in figure 20, case (iii), for which the largest necessary representation is a quadruplet. In order to avoid lower order contributions, one needs the quadruplet $\mathbf{4}_{1/2}^S$ ($\mathbf{4}_{-1/2}^F$) outside the loop. Moreover, one should be able to distinguish between these quadruplets and a Higgs or a lepton doublet. For this reason, the external quadruplet must couple to a singlet and another quadruplet running inside the loop. All the diagrams of this type are depicted in figure 20 and they always contain two quadruplets, one outside the loop and another inside.

As we are dealing with the operator $LLHHHH^\dagger$, the maximum hypercharges of an external quadruplet is $3/2$, i.e. S . These diagrams corresponds to group (ii) defined above. All the diagrams given in figure 19 contains this scalar entering the loop and they belong to the same topology T16 (figure 13). Note that the hypercharge $3/2$ of S prevents the possibility to reduce these models to $d = 5$ 1-loop.

The rest of the diagrams do not contain a external quadruplet. In the minimal case, all the diagrams given in figure 18 just need one quadruplet running in the loop. Diagrams generated from the topologies T2, T12 and T13 with a triplet entering the loop have all similar structures to those given in figure 8. The minimum representations for the fields (χ, η) or (η_1, η_2) in figure 8, are then a singlet and a quadruplet, in order to prevent a coupling of these fields with a lepton doublet L or the Higgs H , respectively.

The remaining diagram T10-i of figure 18 is a rather singular case of this group (i), with only one internal quadruplet. Given the isolated H^\dagger and the upper asymmetric structure with three Higgses, the representation between the Higgs vertices needs to be (at least) a quadruplet, otherwise a $d = 5$ 1-loop contribution is possible.

After giving all the diagrams that can be construct with quadruplets as the highest representation (figures 18–20), the last case depicted in figure 9 shows the only genuine

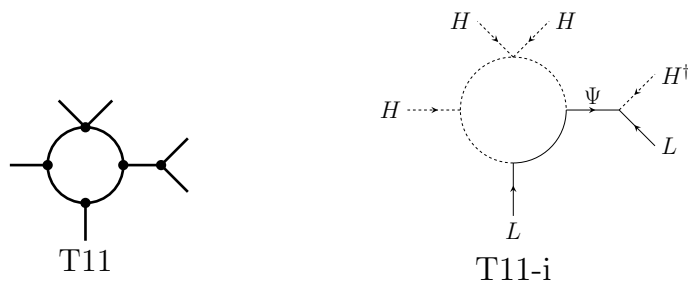


Figure 9. Topology T11 to the left: the only topology for which a genuine model with no representations larger than triplets exist. The only genuine diagram for this topology is shown on the right.

diagram that can be constructed with no representation bigger than triplet. Despite its similarity to the structure given in figure 8 (left), the 4-leg vertex prevents lower order neutrino masses already with triplets. The corresponding diagram figure 9 (right) with a 4-legs vertex followed by a triplet Ψ cannot be bridged to construct a 1-loop $d = 5$ contribution given the relation between the hypercharges of the fields running inside then loop.

To summarize, from the 8 genuine topologies, one can generate 23 diagrams. Among them, only one can be realized with no representation larger than triplets as the maximum $SU(2)_L$ representation, figure 9. The other 22 diagrams generated from the 7 topologies given in figure 13 can generate models with representations up to quadruplets. This whole set can be divided depending if the diagrams require one quadruplet running in the loop (figure 18), outside the loop (figure 19) or two quadruplets both inside and outside the loop (figure 20). Of course, models with larger representations can be constructed and we will give one example in the next subsection.

3.3 Example models

The complete list of diagrams, from which genuine $d = 7$ 1-loop models can be built is given in the appendix. Here, we will briefly discuss three example models, which are among the most simple models one can built from these diagrams. These models are: (i) The simplest $d = 7$ model, which requires no representation larger than a triplet; (ii) one example model with an external quadruplet S ; and (iii) an example model with an $SU(2)_L$ quintuplet. The latter serves to show, how models with larger representations can easily be constructed from our list of diagrams.

3.3.1 Triplet model

As discussed above, there is only one possible diagram that has a triplet as the largest $SU(2)_L$ representation, see figure 9. The model requires the fermionic triplet $\Psi = \mathbf{3}_1^F$, that also appears in the BNT model. A priori, for the particles inside the loop hypercharge is not fixed. However, not all choices of hypercharge will lead to genuine models, since lower order contributions might appear. If we use only doublets and triplets inside the loop, the

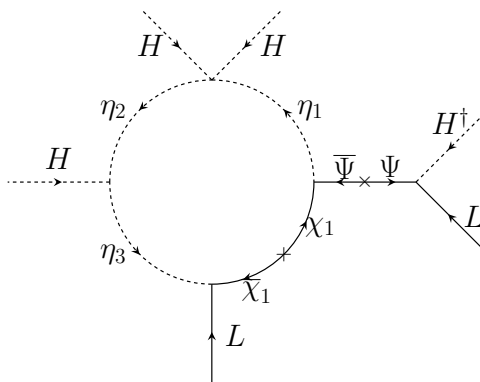


Figure 10. The most minimal model that one can construct at 1-loop $d = 7$ order with no $SU(2)_L$ representations larger than triplet. The model is generated from the diagram T11-i in figure 9.

smallest hypercharge assignments that lead to a genuine model are:

$$\begin{aligned} \Psi &= \begin{pmatrix} \Psi^{++} \\ \Psi^+ \\ \Psi^0 \end{pmatrix} \sim \mathbf{3}_1^F & \eta_1 &= \begin{pmatrix} \eta_1^{++} \\ \eta_1^+ \end{pmatrix} \sim \mathbf{2}_{3/2}^S & \eta_2 &= \begin{pmatrix} \eta_2^{+++} \\ \eta_2^{++} \end{pmatrix} \sim \mathbf{2}_{5/2}^S \\ \eta_3 &= \begin{pmatrix} \eta_3^{++++} \\ \eta_3^{+++} \\ \eta_3^{++} \end{pmatrix} \sim \mathbf{3}_3^S & \chi_1 &= \begin{pmatrix} \chi_1^{++++} \\ \chi_1^{+++} \end{pmatrix} \sim \mathbf{2}_{5/2}^F. \end{aligned}$$

The model generates neutrino masses via the diagram depicted in figure 10. η_1 has the smallest hypercharge of the particles in the loop. For colorless particles it is not possible to find a smaller hypercharge assignment that leads to a genuine model. For example, choosing $\eta_1 = \mathbf{2}_{1/2}^S$ instead would result in a model, which also has the diagram T-3 at $d = 5$ level.

One interesting aspect of this model is that the scalar triplet inside the loop has a component which carries 4 units of electric charge.

3.3.2 Quadruplet model

While for triplets as the maximal representation there is only one diagram, for quadruplets three distinct groups of model exist, as discussed above. We choose an example based on an external quadruplet $S = \mathbf{4}_{3/2}^S$. The example model we choose is based on diagram T16-ii.

As in the triplet case, hypercharge and $SU(2)_L$ representation are not uniquely fixed. The minimal model, again in the sense of using the smallest possible hypercharge assignment for colourless internal fields, has the following particle content:

$$\begin{aligned} S &= \begin{pmatrix} S^{+++} \\ S^{++} \\ S^+ \\ S^0 \end{pmatrix} \sim \mathbf{4}_{3/2}^S & \chi_1 &= \begin{pmatrix} \chi_1^{++} \\ \chi_1^+ \end{pmatrix} \sim \mathbf{2}_{3/2}^F & \chi_2 &= \begin{pmatrix} \chi_2^{++++} \\ \chi_2^{+++} \\ \chi_2^{++} \end{pmatrix} \sim \mathbf{3}_3^F \\ \eta_1 &= \eta_1^{++} \sim \mathbf{1}_2^S & \eta_2 &= \begin{pmatrix} \eta_2^{+++} \\ \eta_2^{++} \end{pmatrix} \sim \mathbf{2}_{5/2}^S. \end{aligned}$$

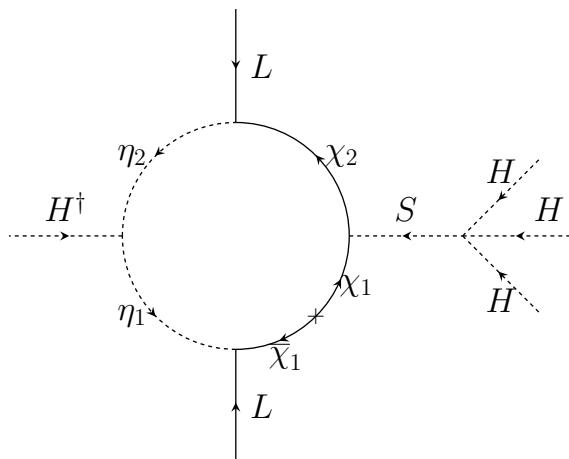


Figure 11. Example of a $d = 7$ 1-loop model with an external quadruplet $S = 4_{3/2}^S$, generated from the diagram T16-ii in figure 19. This model contains only doublet and triplet representations inside the loop, see text.

This model generates neutrino masses via the diagram of figure 11. In this example, lower order contributions can be avoided due to the hypercharge of S . A model with smaller hypercharges, for example η_1 chosen to be $\eta_1 = \mathbf{1}_1^S$, would again not be genuine, since it would necessarily have a $d = 5$ 1-loop contribution, i.e. the classical Zee model [16], see diagram T-1-ii in figure 2. Note that, while the triplet model contains a scalar with 4 units of electric charge, in the quadruplet model it is an internal fermion that has such a large electric charge.

3.3.3 Quintuplet model

Finally, our last example is a model based on diagram T13-i in figure 18. It contains the field $\phi = \mathbf{3}_0^S$ and a quintuplet in the loop. The diagram for the generation of the neutrino masses is shown in figure 12. The minimal particle content containing a 5-plet is given by:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \\ \phi^- \end{pmatrix} \sim \mathbf{3}_0^F \quad \Psi = \begin{pmatrix} \Psi^{++} \\ \Psi^+ \\ \Psi^0 \end{pmatrix} \sim \mathbf{3}_1^F \quad \chi_1 = \begin{pmatrix} \chi_1^{++++} \\ \chi_1^{+++} \\ \chi_1^{++} \\ \chi_1^+ \\ \chi_1^0 \\ \chi_1^- \end{pmatrix} \sim 4_{3/2}^F$$

$$\eta_1 = \begin{pmatrix} \eta_1^{++} \\ \eta_1^+ \end{pmatrix} \sim \mathbf{2}_{3/2}^S \quad \eta_2 = \begin{pmatrix} \eta_2^{+++} \\ \eta_2^{++} \\ \eta_2^+ \\ \eta_2^0 \\ \eta_2^- \end{pmatrix} \sim \mathbf{5}_1^S.$$

The maximum representation in this model is a quintuplet, η_2 . Since it couples to the Higgs, ϕ and η_1 , η_1 could be either a **2**, **4**, **6** or a **8**. However, only for the case of η_1 being a doublet, a genuine model results. This is because, once the coupling $\eta_1 H \eta_2^\dagger$ is allowed, one can construct again a $d = 5$ 1-loop diagram with the particle content of the model.

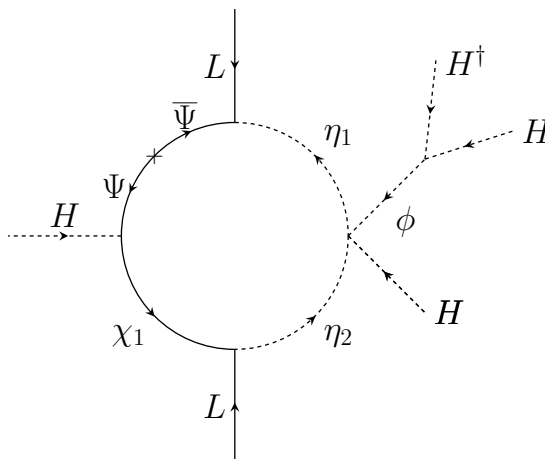


Figure 12. Example of one of the most minimal models that one can construct at $d = 7$ 1-loop order with $SU(2)_L$ representations up to 5-plets generated from the diagram T13-i in figure 18.

It is worth noting that from three example models we have discussed, the quintuplet model is the only one, in which the representations in the loop contain a neutral component. For this model, one can thus follow the idea of the scotogenic model [15]: add a discrete Z_2 symmetry to the model, under which the internal particles are odd, and the lightest neutral particle can be a cold dark matter candidate.

4 Summary

We have discussed neutrino masses at 1-loop $d = 7$ order. We have identified all possible topologies that can lead to genuine models, i.e. models that are not accompanied by either a $d = 5$ or $d = 7$ tree-level mass term nor by a $d = 5$ 1-loop neutrino mass. We have found that only 8 out of a total of 48 topologies can lead to genuine models.

We then ordered the remaining, possibly genuine, diagrams into different groups, depending on the minimal field content necessary to construct a model. There is only one possible diagram for which the largest necessary representation is a triplet. The remaining 7 topologies yield 22 diagrams, with the largest representation being at least a quadruplet. We then briefly discussed three example models, starting from the triplet model, with one additional example for a quadruplet and one for a quintuplet each.

To avoid lower order neutrino masses, the “genuine” models we discussed always have to introduce five new multiplets, usually with quite a large hypercharge for at least one of them. Thus, these $d = 7$ models are necessarily more complicated constructions than the classical seesaw. From a theoretical point of view this might make these models less attractive. However, in particular due to the large electrical charges in these models, one can expect interesting signatures for them at colliders. We reiterate that the $d = 7$ 1-loop contribution can only be dominant, if at least some of the new particles have masses below roughly 2 TeV.

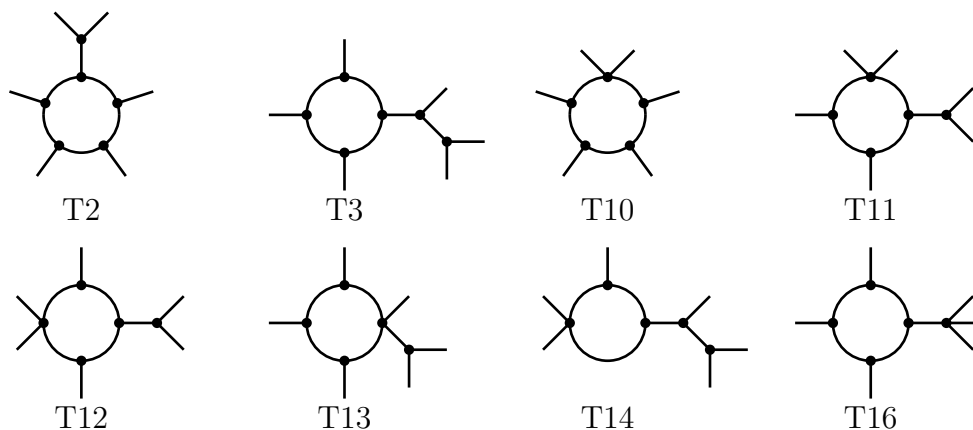


Figure 13. Topologies that can lead to a genuine $d = 7$ 1-loop neutrino mass model. T11 is the only topology for which the largest representation can be as small as a $SU(2)_L$ triplet. For all other topologies at least one quadruplet must appear in the diagram for the model to be genuine. The quadruplet diagrams based on those topologies are shown in figures 18, 19 and 20. For the triplet model see figure 10.

Acknowledgments

We thank R. Fonseca for discussion. This work was supported by the Spanish MICINN grants FPA2014-58183-P and Multidark CSD2009-00064 (MINECO), FPU15/03158 (MECD) and PROMETEOII/2014/084 (Generalitat Valenciana). J.C.H. is supported by Chile grants Fondecyt No. 1161463, Conicyt ACT 1406 and Basal FB0821.

A Topologies and genuine diagrams

In this appendix we present the list of all $d = 7$ 1-loop topologies, classified into genuine and non-genuine topologies, as discussed in the main text. We also give the complete list of diagrams that can lead to “genuine” $d = 7$ neutrino mass models with $SU(2)_L$ quadruplet representations.

A.1 Topologies

Figure 13 shows the 8 topologies that *can* lead to genuine $d = 7$ 1-loop models. We stress again that not all diagrams, derived from these topologies, are necessarily genuine, as discussed in the main text. Note that only T11 can give a model in which the largest representation can be as small as a $SU(2)_L$ triplet. All other 7 topologies require at least one quadruplet for genuine models.

In figure 14 we list all topologies for which *all* diagrams are excluded, since they contain either a singlet fermion ν_R ($\mathbf{1}_0^F$) or a triplet scalar Δ ($\mathbf{3}_{-1}^S$) or a triplet fermion Σ ($\mathbf{3}_0^F$). All diagrams from these topologies thus will also generate a tree-level $d = 5$ seesaw contribution to the neutrino mass matrix.

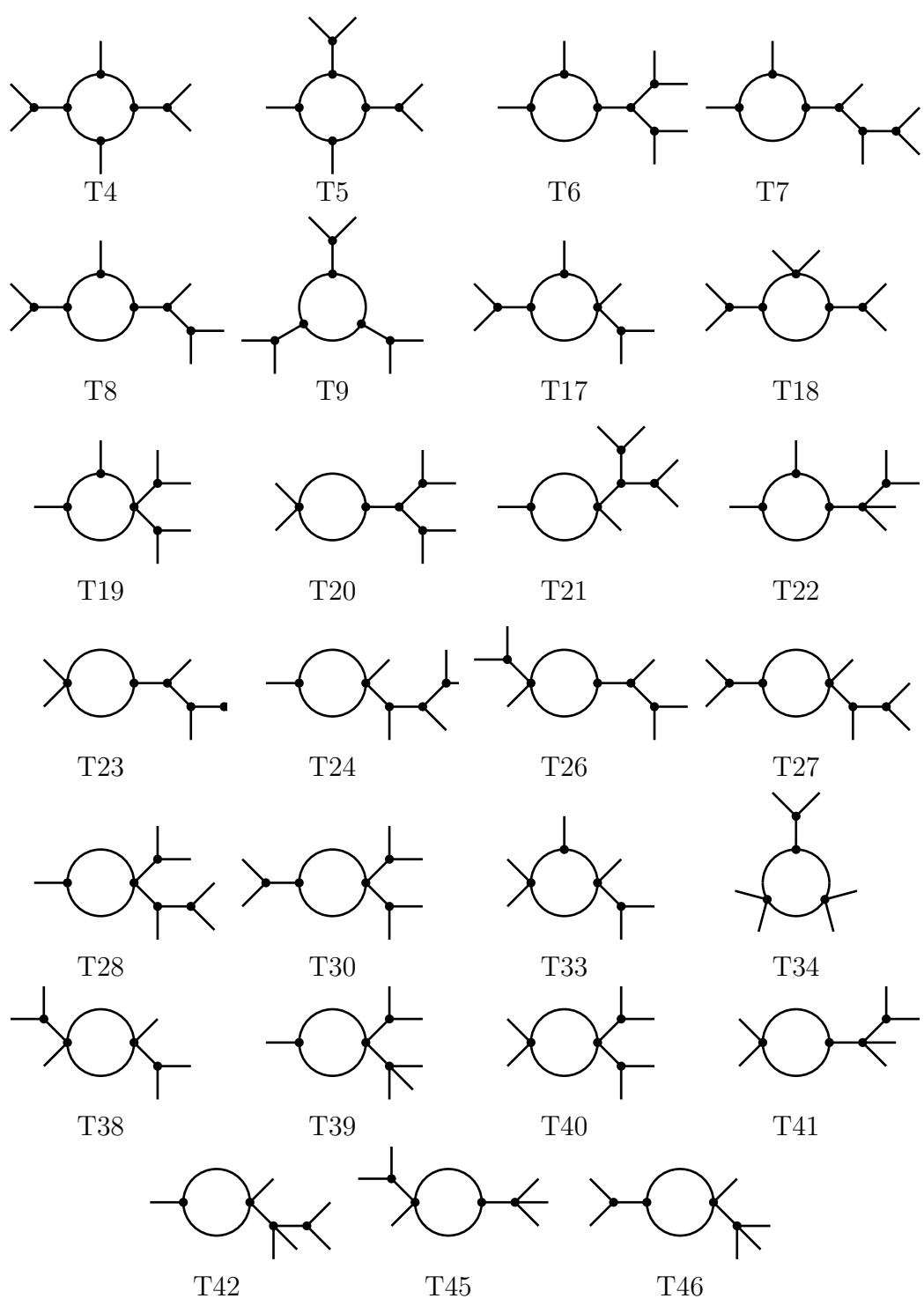


Figure 14. Topologies that necessarily lead to a $d = 5$ tree level neutrino mass, see text.

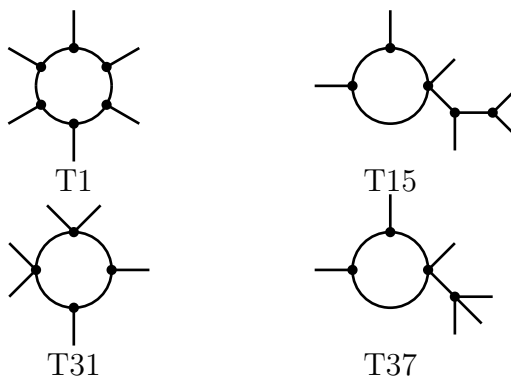


Figure 15. Topologies that lead to a $d = 5$ 1-loop neutrino mass. All diagrams generated from the topologies in this class, not already excluded because they generate a $d = 5$ tree-level mass, include the particle content necessary to generate neutrino mass through one of the four genuine $d = 5$ 1-loop diagrams, see figure 2. Note that T15 is an exceptional case, since it has diagrams for all three possibilities: tree-level $d = 5$, 1-loop $d = 5$ and tree-level $d = 7$.

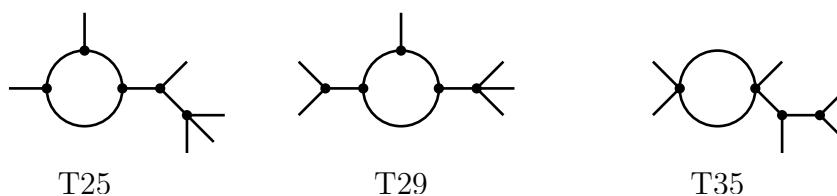


Figure 16. Topologies, which lead to a $d = 7$ tree level neutrino mass. For each of these topologies one can construct diagrams, which have a $d = 5$ tree-level mass. All remaining diagrams contain the scalar S ($\mathbf{4}_{3/2}^S$) along with the fermion Ψ ($\mathbf{3}_1^F$) (figure 1) [30].

In figure 15 we list the topologies for which many but not all diagrams are excluded by a $d = 5$ tree-level seesaw. For these topologies *all remaining* diagrams are excluded because a 1-loop $d = 5$ contribution to the neutrino mass necessarily exists.

In figure 16 we list topologies, which lead to a $d = 7$ tree level neutrino mass. For each of these topologies one can construct diagrams, which have a $d = 5$ tree-level mass. All remaining diagrams, contain the scalar S ($\mathbf{4}_{3/2}^S$) along with the fermion Ψ ($\mathbf{3}_1^F$) and thus generate the $d = 7$ tree-level BNT model [30]. We note in passing that one can, in principle, use these diagrams to radiatively generate one of the vertices in the BNT model. This is very similar to the discussion for the radiative generation of a seesaw coupling given in [8] at $d = 5$ level.

In figure 17 for completeness we show the topologies which are excluded, since they can never lead to a renormalizable model.

A.2 Genuine diagrams

In this section we list diagrams with quadruplets. All diagrams are given in figures 18, 19 and 20. We have divided these diagrams into three groups, depending on whether there is a quadruplet in the loop (figure 18), the scalar S on the outside of the loop (figure 19) or models with at least two different quadruplets (figure 20).

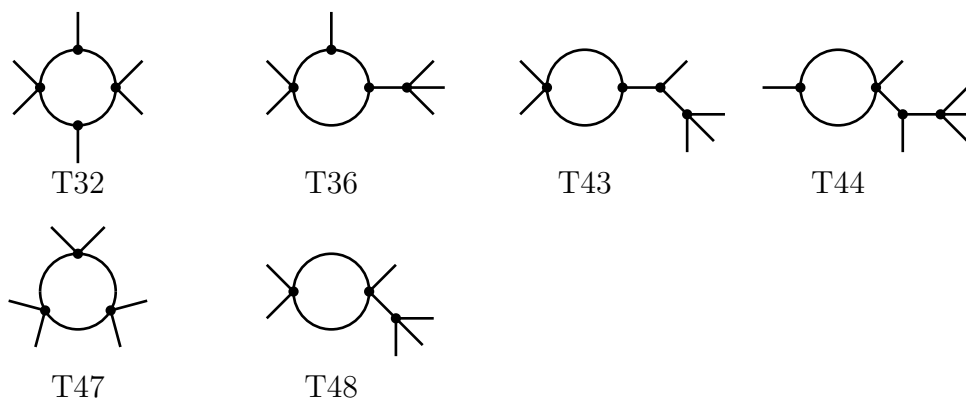


Figure 17. Topologies discarded because they lead to non-renormalizable operators.

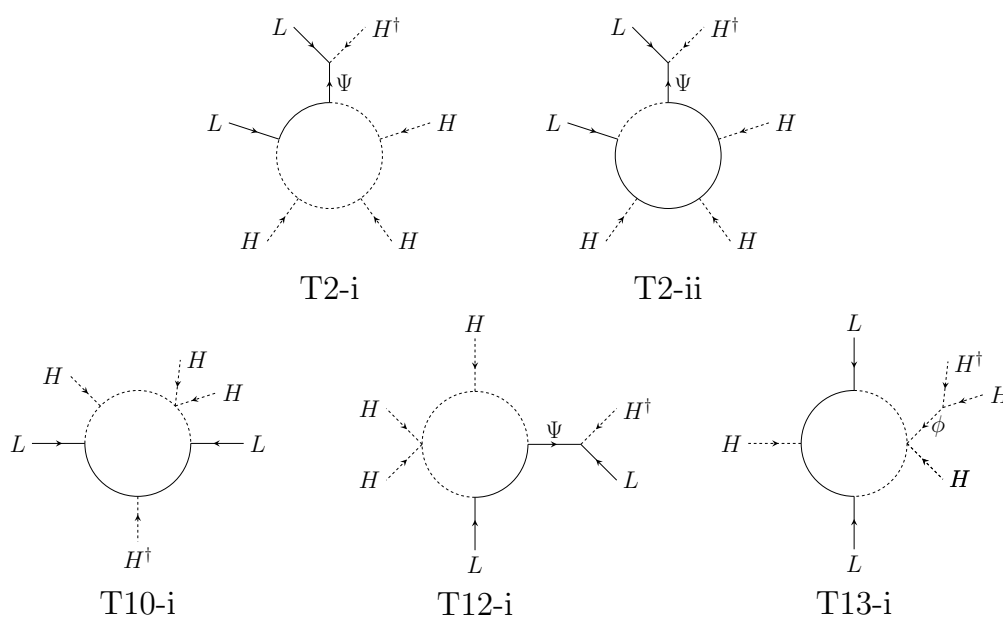


Figure 18. Diagrams that can lead to a genuine $d = 7$ 1-loop neutrino mass for which the largest representations of $SU(2)_L$ is at least a quadruplet. This group of diagrams require the quadruplet to be one of the particles inside the loop to avoid lower order contributions.

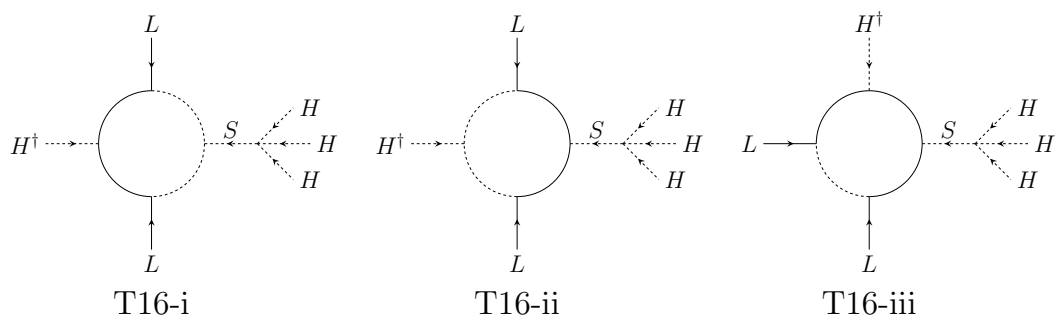


Figure 19. Diagrams that lead to a genuine $d = 7$ 1-loop neutrino mass for which the largest representations of $SU(2)_L$ is at least a quadruplet. All these diagrams contain $S = 4_{3/2}^S$. The hypercharge of the scalar S ensures the absence of a $d = 5$ 1-loop neutrino mass.

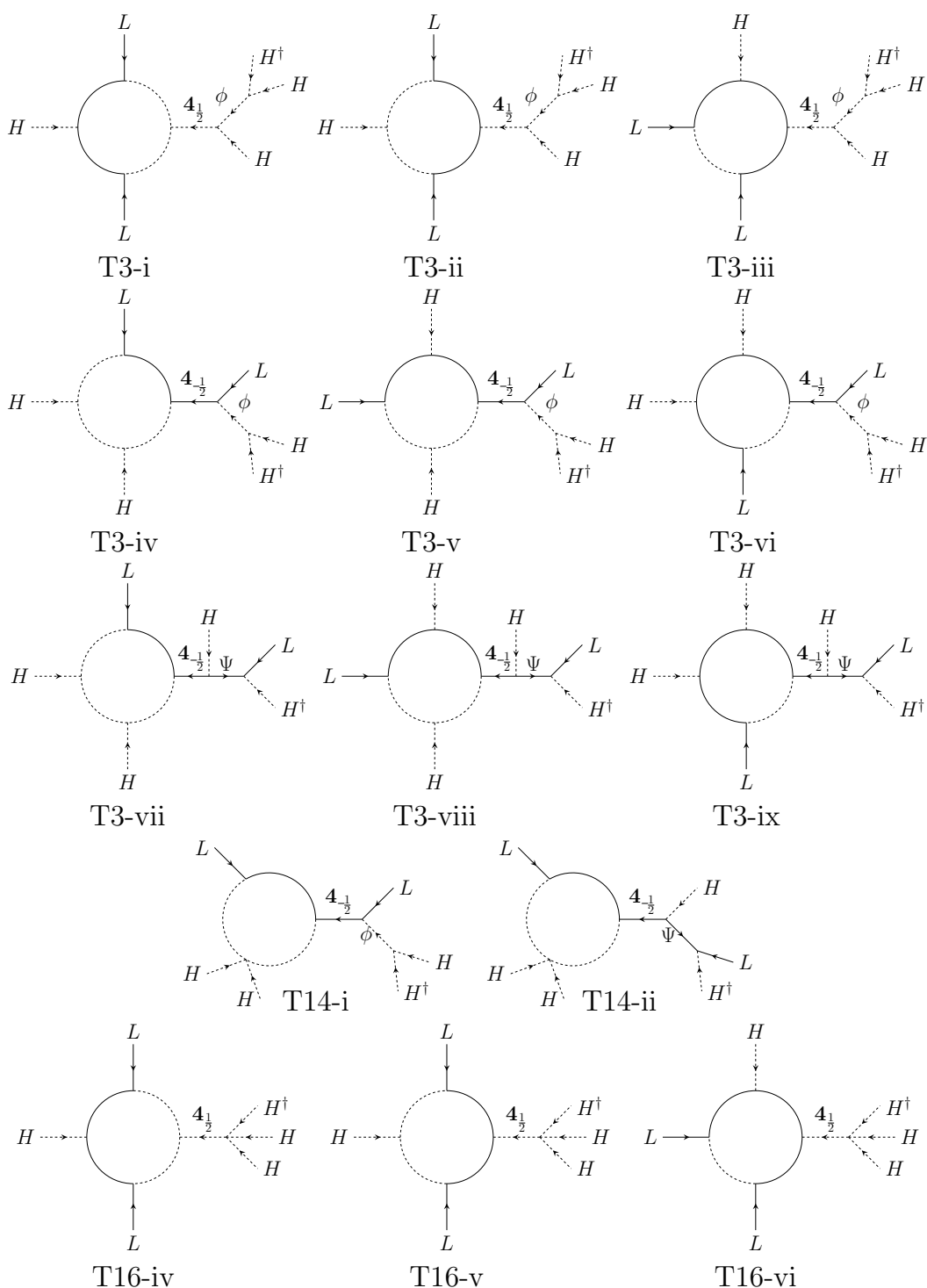


Figure 20. All remaining diagrams that lead to a genuine $d = 7$ 1-loop neutrino mass for which the maximum representations of $SU(2)_L$ is at least a quadruplet. In these diagrams, two quadruplets are needed. Along with the external fermion or scalar quadruplet, a genuine models needs an internal quadruplet to allow to distinguish between a $4_{1/2}^S$ ($4_{-1/2}^F$) and a Higgs (L).

Open Access. This article is distributed under the terms of the Creative Commons Attribution License ([CC-BY 4.0](https://creativecommons.org/licenses/by/4.0/)), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

References

- [1] S. Weinberg, *Baryon and Lepton Nonconserving Processes*, *Phys. Rev. Lett.* **43** (1979) 1566 [[INSPIRE](#)].
- [2] H.K. Dreiner, *An introduction to explicit R-parity violation*, [hep-ph/9707435](#) [[INSPIRE](#)].
- [3] M. Hirsch, M.A. Diaz, W. Porod, J.C. Romao and J.W.F. Valle, *Neutrino masses and mixings from supersymmetry with bilinear R parity violation: A theory for solar and atmospheric neutrino oscillations*, *Phys. Rev. D* **62** (2000) 113008 [*Erratum ibid.* **D 65** (2002) 119901] [[hep-ph/0004115](#)] [[INSPIRE](#)].
- [4] R.N. Mohapatra and J.W.F. Valle, *Neutrino Mass and Baryon Number Nonconservation in Superstring Models*, *Phys. Rev. D* **34** (1986) 1642 [[INSPIRE](#)].
- [5] E.K. Akhmedov, M. Lindner, E. Schnapka and J.W.F. Valle, *Left-right symmetry breaking in NJLS approach*, *Phys. Lett. B* **368** (1996) 270 [[hep-ph/9507275](#)] [[INSPIRE](#)].
- [6] E.K. Akhmedov, M. Lindner, E. Schnapka and J.W.F. Valle, *Dynamical left-right symmetry breaking*, *Phys. Rev. D* **53** (1996) 2752 [[hep-ph/9509255](#)] [[INSPIRE](#)].
- [7] E. Ma, *Pathways to naturally small neutrino masses*, *Phys. Rev. Lett.* **81** (1998) 1171 [[hep-ph/9805219](#)] [[INSPIRE](#)].
- [8] F. Bonnet, M. Hirsch, T. Ota and W. Winter, *Systematic study of the $D = 5$ Weinberg operator at one-loop order*, *JHEP* **07** (2012) 153 [[arXiv:1204.5862](#)] [[INSPIRE](#)].
- [9] D. Aristizabal Sierra, A. Degee, L. Dorame and M. Hirsch, *Systematic classification of two-loop realizations of the Weinberg operator*, *JHEP* **03** (2015) 040 [[arXiv:1411.7038](#)] [[INSPIRE](#)].
- [10] Y. Farzan, S. Pascoli and M.A. Schmidt, *Recipes and Ingredients for Neutrino Mass at Loop Level*, *JHEP* **03** (2013) 107 [[arXiv:1208.2732](#)] [[INSPIRE](#)].
- [11] K.S. Babu and C.N. Leung, *Classification of effective neutrino mass operators*, *Nucl. Phys. B* **619** (2001) 667 [[hep-ph/0106054](#)] [[INSPIRE](#)].
- [12] F. del Aguila, A. Aparici, S. Bhattacharya, A. Santamaria and J. Wudka, *Effective Lagrangian approach to neutrinoless double beta decay and neutrino masses*, *JHEP* **06** (2012) 146 [[arXiv:1204.5986](#)] [[INSPIRE](#)].
- [13] J.C. Helo, M. Hirsch and T. Ota, *Long-range contributions to double beta decay revisited*, *JHEP* **06** (2016) 006 [[arXiv:1602.03362](#)] [[INSPIRE](#)].
- [14] F. Bonnet, D. Hernandez, T. Ota and W. Winter, *Neutrino masses from higher than $D = 5$ effective operators*, *JHEP* **10** (2009) 076 [[arXiv:0907.3143](#)] [[INSPIRE](#)].
- [15] E. Ma, *Verifiable radiative seesaw mechanism of neutrino mass and dark matter*, *Phys. Rev. D* **73** (2006) 077301 [[hep-ph/0601225](#)] [[INSPIRE](#)].
- [16] A. Zee, *A Theory of Lepton Number Violation, Neutrino Majorana Mass and Oscillation*, *Phys. Lett. B* **93** (1980) 389 [*Erratum ibid.* **B 95** (1980) 461] [[INSPIRE](#)].

- [17] M.-C. Chen, A. de Gouvêa and B.A. Dobrescu, *Gauge Trimming of Neutrino Masses*, *Phys. Rev. D* **75** (2007) 055009 [[hep-ph/0612017](#)] [[INSPIRE](#)].
- [18] S. Kanemura and T. Ota, *Neutrino Masses from Loop-induced $d \geq 7$ Operators*, *Phys. Lett. B* **694** (2011) 233 [[arXiv:1009.3845](#)] [[INSPIRE](#)].
- [19] M.B. Krauss, T. Ota, W. Porod and W. Winter, *Neutrino mass from higher than $D = 5$ effective operators in SUSY and its test at the LHC*, *Phys. Rev. D* **84** (2011) 115023 [[arXiv:1109.4636](#)] [[INSPIRE](#)].
- [20] M.B. Krauss, D. Meloni, W. Porod and W. Winter, *Neutrino Mass from a $D = 7$ Effective Operator in an SU(5) SUSY-GUT Framework*, *JHEP* **05** (2013) 121 [[arXiv:1301.4221](#)] [[INSPIRE](#)].
- [21] I. Gogoladze, N. Okada and Q. Shafi, *NMSSM and Seesaw Physics at LHC*, *Phys. Lett. B* **672** (2009) 235 [[arXiv:0809.0703](#)] [[INSPIRE](#)].
- [22] P.-H. Gu, H.-J. He, U. Sarkar and X.-m. Zhang, *Double Type-II Seesaw, Baryon Asymmetry and Dark Matter for Cosmic e^\pm Excesses*, *Phys. Rev. D* **80** (2009) 053004 [[arXiv:0906.0442](#)] [[INSPIRE](#)].
- [23] P. Minkowski, *$\mu \rightarrow e\gamma$ at a Rate of One Out of 10^9 Muon Decays?*, *Phys. Lett. B* **67** (1977) 421 [[INSPIRE](#)].
- [24] T. Yanagida, *Horizontal Symmetry And Masses Of Neutrinos*, *Conf. Proc. C* **7902131** (1979) 95 [[INSPIRE](#)].
- [25] R.N. Mohapatra and G. Senjanović, *Neutrino Mass and Spontaneous Parity Violation*, *Phys. Rev. Lett.* **44** (1980) 912 [[INSPIRE](#)].
- [26] J. Schechter and J.W.F. Valle, *Neutrino Masses in $SU(2) \times U(1)$ Theories*, *Phys. Rev. D* **22** (1980) 2227 [[INSPIRE](#)].
- [27] R.N. Mohapatra and G. Senjanović, *Neutrino Masses and Mixings in Gauge Models with Spontaneous Parity Violation*, *Phys. Rev. D* **23** (1981) 165 [[INSPIRE](#)].
- [28] T.P. Cheng and L.-F. Li, *Neutrino Masses, Mixings and Oscillations in $SU(2) \times U(1)$ Models of Electroweak Interactions*, *Phys. Rev. D* **22** (1980) 2860 [[INSPIRE](#)].
- [29] R. Foot, H. Lew, X.G. He and G.C. Joshi, *Seesaw Neutrino Masses Induced by a Triplet of Leptons*, *Z. Phys. C* **44** (1989) 441 [[INSPIRE](#)].
- [30] K.S. Babu, S. Nandi and Z. Tavartkiladze, *New Mechanism for Neutrino Mass Generation and Triply Charged Higgs Bosons at the LHC*, *Phys. Rev. D* **80** (2009) 071702 [[arXiv:0905.2710](#)] [[INSPIRE](#)].
- [31] Y. Liao, G.-Z. Ning and L. Ren, *Flavor Violating Transitions of Charged Leptons from a Seesaw Mechanism of Dimension Seven*, *Phys. Rev. D* **82** (2010) 113003 [[arXiv:1008.0117](#)] [[INSPIRE](#)].
- [32] S. Fraser, C. Kownacki, E. Ma and O. Popov, *Type II Radiative Seesaw Model of Neutrino Mass with Dark Matter*, *Phys. Rev. D* **93** (2016) 013021 [[arXiv:1511.06375](#)] [[INSPIRE](#)].