Rescaling of quantized skyrmions: from nucleon to baryons with heavy flavor

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Abstract

The role of rescaling (expansion or squeezing) of quantized skyrmions is studied for the spectrum of baryons beginning with nucleon and $\Delta(1232)$, and with flavors strangeness, charm or beauty. The expansion of skyrmions due to the centrifugal forces has influence on the masses of baryons without flavor (N and especially Δ). The rescaling of skyrmions has smaller influence on the spectrum of strange baryons, it is more important for the case of charm, and is crucial for baryons with beauty quantum number, where strong squeezing takes place. Two competing tendencies are clearly observed: expansion of skyrmions when isospin (or spin) increases, and squeezing with increasing mass of the flavor. For the case of beauty baryon Λ_b satisfactory agreement with data can be reached for the value $r_b = F_B/F_\pi \simeq 2.6$, for the case of Σ_b there should be $r_b \sim 2$, so for the beauty flavor the method seems to be not quite satisfactory because of certain intrinsic discrepances. Some pentaquark states with hidden strangeness, charm or beauty are considered as well.

1 Introduction

Studies of baryons spectrum is one of important directions of the elementary particle physics, and the chiral soliton approach [1, 2] provides one of attrctive possibilities for this, after different kinds of the quark models. The pioneer papers are well known [3, 4], where static properties of baryons (nucleon and $\Delta(1232)$) have been calculated within accuracy about 30% of experimental values.

Two parameters of the model - pion decay constant F_π and the Skyrme constant e were defined in [3, 4] by fitting the masses of the nucleon and $\Delta(1232)$ which allowed to calculate other static properties of these baryons. The classicaal mass of the soliton, skyrmion with baryon (winding) number $B = 1$ - was obtained by direct minimization of the static energy functional. The energy (mass) of the quantized states (baryons) is the sum of the classical mass and isospin dependent quantum correction. The chiral fields configuration for each of quantized states does not satisfy the Euler-Lagrange equation, and the sum of the classical mass of the skyrmion plus quantum correction could be minimized further, but this issue has not been discussed in [3, 4] because the quantum correction turned out to be small enough for the nucleon — less than $\sim 10\%$, although greater and much more important for the $\Delta(1232)$, because it is responsible for the mass splitting between nucleon and Δ.

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The $SU(3)$ extention of the model has been proposed somewhat later in [5] which allowed to calculate the mass splittings between components of the $SU(3)$ multiplets of baryons. The $SU(3)$ violating mass term, added to the model lagrangian, was considered as a small enough perturbation, similar to the quantum correction in the $SU(2)$ model [3], although in some cases this correction is even greater. Therefore, the problem that the energy of the quantized state is not the minimal energy, appears here again, even in greater scale.

In present paper we investigate this problem using the simple variant of the quantization scheme, proposed by Klebanov and Westerberg [6] and slightly modified in [7, 8]. We study the influence of the change of the scale (dimension) of the whole skyrmion on the energy of the quantized states with heavy flavors, and with strangeness as well. The squeezing of the skyrmion leads to considerable decrease of the energy (mass) of the quantized state, which is especially important for the charm or beauty quantum numbers, but not so important for the case of strangeness. For the quantized states with u, d flavors only (but not strangeness, or charm, or beauty) the expansion of the state due to the centrifugal forces takes place, instead of squeezing.

Features of the chiral soliton approach are described briefly in the next section, where some static characteristics of the skyrmion are presented. The quantization scheme is described in section 3, where the moments of inertia of the skyrmion and flavor excitation energies are given as well. The spectrum of baryon states is presented in section 4. The masses of some positive parity pentaquarks with hidden flavor (strangeness, or charm, or beauty) are estimated in section 5 within same approach. Final section contains discussion and some conclusions.

2 Features of the chiral soliton approach and some static properties of the skyrmion

The starting point of the CSA, as well as of the chiral perturbation theory, is the effective chiral lagrangian written in terms of the chiral fields incorporated into the unitary matrix $U \in SU(2)$ in the original variant of the model [1, 2], $U = cos\, f + isin\, f\ \vec{\tau}\vec{n}$, $n_z = cos\alpha$, $n_x = sin\alpha\, cos\beta$, $n_y = sin\alpha\ \sin\beta$, where functions f (the profile of the skyrmion), and angular functions α, β in general case are the functions of 3 coordinates x, y, z. To get the states with flavor s, c or b we make extension of the basic $U \in SU(2)$ to $U \in SU(3)$ with (u, d, s), (u, d, c) or $((u, d, b)$ degrees of freedom.

It is convenient to write the lagrangian density of the model in terms of left (or right) chiral derivative

$$l_\mu = \partial_\mu U U^\dagger = -U\partial_\mu U^\dagger \tag{1}$$

$$\mathcal{L} = -\frac{F_\pi^2}{16} l_\rho l_\rho + \frac{1}{e^2}\left[l_\rho l_\tau\right]^2 + \frac{F_\pi^2 m_\pi^2}{16} Tr(U + U^\dagger - 2) \tag{2}$$

F_π is the pion decay constant, its experimental value is now $F_\pi \simeq 185\, MeV$ [9]; e is the constant introduced by Skyrme [1]. It can be defined experimentally as well, but the allowed interval for this parameter is wide enough presently. Meson properties - mass, decay constants - are input of the model, and baryon properties are deduced from meson properties, according to [1, 2, 3]. This, original variant of the model [1], where soliton stabilization takes place due to the 4-th order term in the lagrangian density (2), is called now the $SK4$ variant.

Mass splittings within $SU(3)$ multiplets of baryons are due to the term in the lagrangian [10], see also [7]:

$$\mathcal{L}^{br} = \frac{F_\pi^2 \widetilde{m}_D^2}{24} Tr(1 - \sqrt{3}\lambda_8)\left(U + U^\dagger - 2\right) + \frac{F_D^2 - F_\pi^2}{48} Tr(1 - \sqrt{3}\lambda_8)\left(U l_\mu l^\mu + l_\mu l^\mu U^\dagger\right), \tag{3}$$

λ_8 is the $SU(3)$ Gell-Mann matrix, $\tilde{m}_D^2 = F_D^2 m_D^2/F_\pi^2 - m_\pi^2$ includes the $SU(3)$-symmetry violation in flavor decay constants, as well as in meson masses. m_D denotes the mass of the kaon, D-meson or B-meson, for strangeness, charm, or beauty, similar holds for F_D.

In this section we present some static properties of the skyrmion which are necessary to perform the procedure of the $SU(3)$ quantization and to obtain the spectrum of states with definite quantum numbers. The quantity Γ, proportional to the sigma-term,

$$\Gamma(\lambda) \simeq \lambda^3 \frac{F_\pi^2}{2} \int (1 - c_f) d^3 r \tag{4}$$

plays important role in any of known quantization modela, in rigid (or soft) rotator model, and in the bound state model, which simplified version we exploit here. The scaling properties of this quantity (i.e. the behaviour under change of the dimension of the soliton $r \to \lambda r$) are shown, which will be important in our consideration. Numerically, for the baryon number $B = 1$ configuration, $\Gamma(\lambda = 1) \sim 5\,Gev^{-1}$. The moments of inertia of skyrmion, the isotopical $\Theta_I \sim (5-6)Gev^{-1}$, the flavor $\Theta_F \sim (2-3)Gev^{-1}$ play an important role as well, see e.g. [7, 8], and references here. All moments of inertia $\Theta \sim N_c$, where N_c is the number of colors of underlying QCD [2]. Expressions for the moments of inertia will be given in the next section.

One of main advantages of the CSA [1] consists in the possibility to consider baryonic states with different flavors - strange, charmed or beautiful - and with different atomic (baryon) numbers from unique point of view, using one and the same set of the model parameters. The properties of the system are evaluated as a function of external quantum numbers which characterize the system as a whole, whereas the hadronic content of the state plays a secondary role. This is in close correspondence with standard experimental situation where e.g. in the missing mass experiments the spectrum of states is measured at fixed external quantum numbers - strangeness or other flavor, isospin, etc. The so called deeply bound antikaon-nuclei states have been considered from this point of view in [11] not in condratiction with data (this is probably one of most striking examples).

Remarkably that the moments of inertia of skyrmions carry information about their interactions. Probably, the first example how it works are the moments of inertia of the toroidal $B = 2$ biskyrmion. The orbital moment of inertia Θ_J is greater than the isotopic moment of inertia Θ_I, as a result, the quantized state with the isospin $I = 0$ and spin $J = 1$ (analogue of the deuteron) has smaller energy than the state with $I = 1,\ J = 0$ (quasi-deuteron, or nucleon-nucleon scattering state), in qualitative agreement with experimental observation that deuteron is bound stronger.

In the pioneer paper [3] the masses of the nucleon and Δ isobar have been fitted, and as a result the pion decay constant turned out to be considerably lower than experimental value $F_\pi \simeq 185\,MeV$. Later another approach has been developed, in particular by Siegen University theory group (G.Holzwarth, B.Schwesinger, H.Walliser and H.Weigel). The idea is that the value of the classical mass of the skyrmion is controlled by poorly known loop corrections of the order of N_c^0, or so called Casimir energy, [12]. Therefore, it makes more sense to calculate the differences of masses of baryons with different quantum numbers, like difference of masses of nucleon and $\Delta(1232)$ isobar (as it was made first in [3]), nucleon and hyperons, i.e. mass splittings inside $SU(3)$ multiplets of baryons, calculated first in [5].

The classical mass of the skyrmion is calculated usually with the pion mass term included to the lagrangian, and consists of 3 parts which scale differently:

$$M_{cl} = m_1 \lambda + m_{-1}/\lambda + m_3 \lambda^3 \tag{5}$$

[1] It is the authors opinion, probably, not accepted widely.

with

$$m_1 = F_\pi^2 \frac{\pi}{2} \int \left(f'^2 + 2\frac{s_f^2}{r^2} \right) r^2 dr, \quad m_{-1} = \frac{2\pi}{e^2} \int \frac{s_f^2}{r^2} \left(2f'^2 + \frac{s_f^2}{r^2} \right) r^2 dr,$$

$$m_3 = \pi F_\pi^2 m_\pi^2 \int (1 - c_f) r^2 dr = \frac{m_\pi^2}{2} \Gamma, \tag{5a}$$

which satisfy the Derrick relation

$$m_1 + 3 m_3 = m_{-1}$$

see table 1.

3 Rigid oscillator quantization model, moments of inertia of the skyrmion

We shall use the following mass formula for the quantized state derived in [6] for the quantization scheme used here

$$M(B = 1, F, I, J) = M_{cl} + |F| \omega_F + \Delta E_{1/N_c}. \tag{6}$$

The flavor (antiflavor) excitation energies are

$$\omega_F = \frac{3}{8\Theta_F}(\mu_F - 1); \quad \bar{\omega}_F = \frac{3}{8\Theta_F}(\mu_F + 1) \tag{7}$$

with

$$\mu_F = \left[1 + \frac{16[\bar{m}_D^2 \Gamma + (F_D^2 - F_\pi^2)\widetilde{\Gamma}]\Theta_F}{9} \right]^{1/2} \tag{8}$$

$$\widetilde{\Gamma} = \frac{1}{4} \int c_f \left[c_f (\vec{\partial} f)^2 + s_f^2 (\vec{\partial} n_i)^2 \right] d^3 r, \tag{9}$$

see [7, 8]. Evidently, $\widetilde{\Gamma} \sim \lambda$ under scaling procedure.

Different terms in (6) scale differently as the number of colors in this expression:

$$M_{cl} \sim N_c, \quad \omega_F \sim N_c^0,$$

all moments of inertia $\Theta \sim N_c$.

Previously estimates of the flavor excitation energies were made mostly in perturbation theory, i.e. the flavor excitation energy has been simply added to the skyrmion energy. This is not justified, however, when the flavor excitation energy is large. Here we include this energy into simplified minimization procedure which is made by means of the change of the soliton dimension (rescaling of the soliton). This procedure takes into account the main degree of freedom of the $B = 1$ skyrmion (hedgehog), and skyrmions given by the rational map anzatz [13] which has been applied successfully to describe some properties of nuclei. As we show here, the rescaling leads to considerable decrease of the energy of states, beginning with the $\Delta(1232)$. Similar (although not the same) modification of the quantized skyrmion was made, in particular, by B.Schwesinger et al [14] to improve the description of strange dibaryon configurations.

The hyperfine splitting correction to the energy of states which is formally of the $1/N_c$ order in the number of colors, has been obtained previously in [6] and reproduced in [7, 8]:

$$\Delta E_{1/N_c} = \frac{1}{2\Theta_I} \left[c_F I_r(I_r + 1) + (1 - c_F) I(I + 1) + (\bar{c}_F - c_F) I_F(I_F + 1) \right] \tag{10}$$

I is the isospin of the state, I_F is the isospin carried by flavored meson (K, D, or $B-$meson, for unit flavor $I_F = 1/2$), I_r can be interpreted as "right" isospin, or isospin of basic non-flavored configuration. The hyperfine splitting constants

$$c_F = 1 - \frac{\Theta_I(\mu_F - 1)}{2\Theta_F \mu_F}, \quad \bar{c}_F = 1 - \frac{\Theta_I(\mu_F - 1)}{\Theta_F \mu_F^2}, \tag{11}$$

This correction is considered usually as small one, but it should be included into the minimization procedure, when isospin I is large. Here we include this correction to the masses for all baryons.

At large enough m_D the expansion can be made

$$\mu_F \simeq \frac{4\bar{m}_D(\Gamma\Theta_F)^{1/2}}{3} + \frac{3}{8\bar{m}_D\Gamma\Theta_F},$$

therefore

$$\omega_F \simeq \frac{1}{2}\bar{m}_D\left(\frac{\Gamma}{\Theta_F}\right)^{1/2} - \frac{3}{8\Theta_F}. \tag{12}$$

Here we take the ratio of decay constants $F_K/F_\pi \simeq 1.197$, $F_D/F_\pi \simeq 1.58$ according to the analysis performed by Rosner and Stone in [9]. Our results presented in this paper suggest that the ratio F_B/F_π should be greater, between 2. and 2.6.

The flavored moment of inertia equals (we added the rescaling factor - some power of the parameter λ to make evident the behaviour under the rescaling procedure $r \to r\lambda$)

$$\Theta_F = \lambda f_1 + \lambda^3 f_3^{(0)} \frac{F_D^2}{F_\pi^2} = \Theta_F^{(0)} + \lambda^3 f_3^{(0)} \left(\frac{F_D^2}{F_\pi^2} - 1\right) \tag{13}$$

with

$$f_1 = \frac{\pi}{2e^2}\int (1-c_F)\left(f'^2 + 2\frac{s_f^2}{r^2}\right) r^2 dr; \qquad f_3^{(0)} = \frac{\pi}{2}F_\pi^2 \int (1-c_F) r^2 dr \tag{14}$$

In the integrands f denotes the profile function of the soliton (skyrmion). Here we show explicitly the dependence of different parts of the inertia on the rescaling parameter λ. In table 1 we present numerical values for $f_1,\ f_3,\ t_1, t_3$ and other quantities used to perform calculations of the masses of quantized states.

There is simple connection between total moment of inertia in the $SK4$ variant of the model, the Θ_F and the sigma-term:

$$\Theta_F^{tot} = \frac{F_D^2}{4F_\pi^2}\Gamma + \Theta_F = \frac{F_D^2}{F_\pi^2} f_3^{(0)} + f_1. \tag{15}$$

Similarly, the isotopic moment of inertia Θ_I within the rational map aproximation can be written as

$$\Theta_I = \lambda\, t_1 + \lambda^3\, t_3 \tag{16}$$

with

$$t_1 = \frac{4\pi}{3}\int \frac{2\, s_f^2}{e^2}\left(f'^2 + B\frac{s_f^2}{r^2}\right) r^2 dr, \qquad t_3 = \frac{2\pi}{3}F_\pi^2 \int s_f^2 r^2 dr. \tag{17}$$

	t_1	f_1	t_3	f_3	Γ	$\widetilde{\Gamma}$	m_1	m_{-1}	m_3
ANW	3.21	–	1.90	–	–	–	432	432	0
AN	3.21	–	1.90	–	–	–	357	471	38
$Siegen$	3.49	0.83	2.07	1.20	4.80	15.6	759	897	46

Table 1. Numerical values of the quantities used in present paper, which have definite scaling behaviour. t_1, t_3, f_1, f_3, Γ, $\widetilde{\Gamma}$ are in GeV^{-1}, m_1, m_2 and m_3 are in MeV. First line corresponds to original parametrization of [3], $F_\pi = 129\,MeV,\ e = 5.45$ [2], 2-d line corresponds to the parametrization of [4], $F_\pi = 108\,MeV,\ e = 4.84$, 3-d line — to parametrization of Siegen University group with $F_\pi = 186\,MeV$, $e = 4.12$.

In the $SK6$ variant of the model the skyrmion stabilization takes place due to the 6-th order term (in chiral derivatives) in the lagrangian density, which is proportional to the baryon number density squared, see e.g. [15]. We shall consider this variant of the model elsewhere.

4 Rescaling of the lowest baryons masses

We are using the following expressions for the masses of baryons:

$$M_N = M_{cl} + \frac{3}{8\Theta_I}; \quad M_\Delta = M_{cl} + \frac{15}{8\Theta_I};$$

$$M_\Lambda = M_{cl} + \frac{3}{8\Theta_I} + \omega_F - \frac{3(\mu_F - 1)}{8\mu_F^2\Theta_F}; \quad M_\Sigma = M_{cl} + \frac{3}{8\Theta_I} + \omega_F - \frac{3(\mu_F - 1)}{8\mu_F^2\Theta_F} + \frac{\mu_F - 1}{2\mu_F\Theta_F}. \tag{19}$$

The flavor inertia $\Theta_F^{(0)}$ is the same for all three flavors, see Eq. (13), but the quantity μ_F and the flavor excitation energy ω_F are different for different flavors.

Description of the masses of strange hyperons Λ_s and Σ_s is not perfect in table 2, because the configuration mixing, i.e. the mixing between the states with same isospin and strangeness, but which belong to different $SU(3)$ multiplets, is not taken into account in our approach. Moreover, after rescaling these states cannot mix, because they have different properties (dimensions, in particular). Satisfactory agreement with data has been obtained in [10] just due to including such mixing into consideration. Improvement of the fit is certainly possible in our case as well, by changing of the model parameters, first of all.

For the case of the non-flavored baryons the expansion of the configuration takes place due to centrifugal forces. Technically it appears from the contribution of the spin-isospin dependent term in the energy of quantized state, which contains the isotopic (or orbital) inertia in the denominator. Both isotopic and orbital inertia are proportional to the scale factor λ, or λ^3, and increase of λ (expansion of the skyrmion) leads to the decrease of energy.

[2] We used the values of the soliton mass and the moment of inertia given in [3] by formulas after Eq. (9).

	λ_{min}	δM	$M_B - M_N$	$(M_B - M_N)_{exp}$
N_{ANW}	1.1343	8.1	–	–
$\Delta(1232)_{ANW}$	1.4982	128	174	293
N_{AN}	1.0978	5.2	–	–
$\Delta(1232)_{AN}$	1.3559	199	182	293
N	1.0539	3.1	–	–
$\Delta(1232)$	1.2247	61	212	293
Λ_s	0.8499	26	247	177
Λ_c	0.5478	136	1432	1347
$\Lambda_b(r = 2.60)$	0.2212	2827	4663	4680
$\Lambda_b(r = 2.65)$	0.2192	2855	4724	4680
Σ_s	1.0221	0.4	370	251
Σ_c	0.8920	10.	1682	1515
$\Sigma_b(r = 2.00)$	0.2551	995	4962	4874
$\Sigma_b(r_b = 2.05)$	0.2512	1057	5049	4874

Table 2. The values of λ_{min} at the minimal total energy (mass) of quantized baryon states. The decrease of masses δM due to the change of λ from 1 to λ_{min} is given in MeV. The difference of masses $M_B - M_N$, theoretical and experimental values, are presented as well (in MeV). The first 2 lines correspond to the parametrization in the chiral symmetry limit, proposed in [3]. Lines 3-4 correspond to the parametrization of [4], the pion mass included into lagrangian. Other calculations were performed taking Siegen parametrization, $F_\pi = 186\,MeV,\ e = 4.12$.

The flavor excitation energies are proportional to the mass $\bar{m}_D$, which is large for charm or beauty, and to $\sqrt{\Gamma} \sim \lambda^{3/2}$, and this explains, why λ_{min} is so small for beauty.

5 Estimates of the masses of pentaquarks with hidden flavor

For the case of pentaquarks with hidden flavor, i.e. containing the pair of quark and antiquark, or the pair of D and $\bar{D}$ (or K and $\bar{K}$, or B and $\bar{B}$) mesons, we take in Eq. (10) $I_r = I$ and $I_F = 0$, and come to the energy (mass) of the state

$$M_{P_F} = M_{cl} + \frac{3\mu_F}{4\Theta_F} + \frac{I(I+1)}{2\Theta_I}. \qquad (20)$$

which we minimize numerically. Some results are shown in table 3.

Tables 2 and 3 illustrate well two competing tendencies for quantized skyrmion states: the squeezing with increazing flavor excitation energy, and expansion due to centrifugal forces which becomes stronger with increasing spin (isospin). For beauty squeezing dominates in all cases considered here.

$B(I=J)$	λ_{min}	δM	$M_B - M_N$
$P_s(1/2)$	1.05771	4.8	956
$P_s(3/2)$	1.2006	58	1173
$P_s(5/2)$	1.3884	231	1449
$P_c(1/2)$	0.7235	86	3257
$P_c(3/2)$	0.9808	0.4	3612
$P_c(5/2)$	1.2700	90	3972
$P_b(1/2, r_b = 2.0)$	0.2798	3828	9073
$P_b(1/2, r_b = 2.1)$	0.2735	3960	9341
$P_b(3/2, r_b = 2.0)$	0.3223	2139	10432
$P_b(3/2, r_b = 2.1)$	0.3150	2837	10734
$P_b(5/2, r_b = 2.0)$	0.4099	1359	12267
$P_b(5/2, r_b = 2.1)$	0.4008	1404	13556

Table 3. The values of λ_{min} at the minimal total energy (mass) of some pentaquark states. The decrease of masses δM due to the change of λ from 1 to λ_{min} is given in MeV, similar to table 2. The difference of masses $M_B - M_N$, theoretical values only, are presented as well (in MeV). Calculations are performed taking parametrization $F_\pi = 186\,MeV,\ e = 4.12$.

The states considered in table 3 have positive parity, as a consequence of the quantization scheme used, and isospin which coincides with the right isospin and equals to spin of the state - because the quantized configuration of fields is of hedgehog type. Pentaquarks with $I = J = 1/2$ could belong to the antidecuplet of corresponding $SU(3)$ group, $(p,q) = (0,3)$, those with $I = j = 3/2$ could belong to the $\{27\}$-plet, $(p,q) = (2,2)$ and pentaquarks with $I = J = 5/2$ could belong to the $\{35\}$-plet with $(p,q) = (4,1)$.

To obtain the masses of pentaquarks predicted by this simplified model, one should add the nucleon mass, $939\,MeV$, to the numbers of the last column of table 3. The hidden strangeness pentaquark states presnted in table 3 have masses by few hundreds of MeV greater than such states discussed previously in connection with the low-lying positive strangeness pentaquark $\Theta^+(1540)$, see e.g. discussion in [16]. For example, $M[P_s(J = 1/2)] = 1895\,MeV$. The hidden charm pentaquark state has the mass by $\sim 100MeV$ greater than the mass of the state observed by the LHCb collaboration, $M(P_c) \simeq 4450\,MeV$: $M[P_c(J = 3/2)] \simeq 4550\,MeV$.

6 Conclusions and prospects

We have demonstrated that considerable decrease of the energy of quantized skyrmion states (baryons) takes place due to change of the skyrmton dimension (rescaling). Even for baryons with (u,d) flavors, nucleon and $\Delta(1232)$ isobar, the expansion of the skyrmion due to centrifugal force decreases the mass splitting between N and Δ considerably, and destroys the fit of masses made in [3] and [4] [3]. This fit could be recovered by some increase of parameters of the model - towards better agreement with data, which demands certain technical work.

The change of the skyrmion dimensions leads to considerable lowering of the energy (mass) of the quantized states with quantum numbers charm or beauty. For strangeness the effect takes

[3] It is not difficult algebraic work to define the parameters of the model F_π and e in the chiral symmetry limit of [3]. It has been obtained in [3] for the soliton mass $M_{cl} = aF_\pi/e = (5M_N - M_\Delta)/4$, and for the mass splitting between $\Delta(1232)$ and nucleon $\Delta_M = M_\Delta - M_N = bF_\pi e^3$, $a = 36.5$, the constant b can be extracted from relation for the moment of inertia presented in [3] after Eq. (9). It follows then immediately, $F_\pi = [M_{cl}^3 \Delta_M/(a^3 b)]^{1/4}$ and $e = [a\Delta_M/(bM_{cl})]^{1/4}$. After rescaling there are no such simple relations, but F_π and e should be somewhat greater, $F_\pi \sim 150\,MeV$.

place as well, but not so important - small enough in most of cases. In our estimates we used simple and transparent variant of the quantization procedure, originally proposed in [6] and modified later in [8]. It is quite obvious that the effect is important in any variant of the quantization scheme. There are two competing tendencies, to decrease the dimension of the skyrmion when the flavor excitation energy becomes large, and to expand the skyrmion due to centrifugal forces, when the spin (or isospin) becomes greater. For large enough spin (isospin) of the state the expansion takes place due to centrifugal force - instead of squeezing. For strangeness this effect dominates already for Σ_s hyperon, see table 2, and takes place for all hidden strangeness pentaquarks, and for hidden charm pentaquark with $J = 5/2$ (table 3).

There is some discrepance in description of masses of beauty baryons Λ_b and Σ_b, which indicates that method itself is limited in its applicability. To describe the mass of the Λ_b baryon the ratio $r_b = F_B/F_\pi$ should be about 2.6 , but to obtain satisfactory description of the mass of the Σ_b baryons there should be $r_b \sim 2.0$.

Modifications and improvemants of the approach, including its fine tuning, seem to be of interest.

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